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Mesh-sensitivity analysis of seismic damage index for reinforced concrete columns

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ABSTRACT

Mesh-sensitivity in evaluating a local tensile-damage-parameter-based seismic damage index for reinforced concrete columns is studied. Multiple finite element meshes for nonlinear seismic analysis are used to examine their effect on the simulation results. Mesh-sensitivity of the damage index is investigated using the simulation results of a reinforced concrete column subjected to cyclic loading. A correction method for the damage index is suggested based on nonlinear regression of volumetric tensile damage ratio data. The modified damage index values are presented in the quasi-static cyclic simulation to show the feasibility of the suggested correction method.

Keywords: mesh-sensitivity; seismic damage index; reinforced concrete column; cyclic loading

1. Introduction

Seismic damage index is widely used to evaluate possible damage of structures due to dynamic loads such as earthquakes and collision. It is particularly important in the application to post-earthquake damage assessment, seismic fragility evaluation, loss estimation and retrofitting of structures. There are various seismic damage indices proposed in terms of strains, curvature, rotation, lateral displacement, dissipated energy, etc. to capture structural damages due to excessive displacement and repetitive deformation of members (Park and Ang 1985, Teran-Gilmore and Jirsa 2005, Sinha and Shiradhonkar 2012, Hadzima-Nyarko *et al.* 2014, Andre, Beale *et al.* 2015). The indices,

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however, sometimes have objectivity problem because of ambiguity in determining their basis parameter values such as ultimate displacement and yield strength (Kang and Lee 2016). To overcome the shortcomings, Kang and Lee (2016) developed a new structural damage index for reinforced concrete columns based on a local tensile damage variable of the Lee and Fenves plastic damage model. Despite the advantages of the new damage index to reflect local stiffness degradation of structural members, the damage index value is expected to be sensitive to finite element mesh density because its evaluation is based on the result of nonlinear finite element simulations.

This paper examines the sensitivity of the damage index to mesh density for a reinforced concrete column model subjected to cyclic loading. Multiple finite element meshes with increasing density are used to investigate their effect on the damage index values calculated from nonlinear finite element simulations. With the simulation results, this paper suggests a correction method for the damage index based on nonlinear regression of volumetric tensile damage ratio data. Feasibility of the correction is discussed by comparing the modified damage index values with those obtained using individual meshes.

2. Damage index based on plastic-damage model

Kang and Lee (2016) have suggested a global damage index based on the plastic-damage model by Lee and Fenves (1998a, 1998b). In this section the formulation of the damage index is outlined.

In the plastic-damage model, the stress $\boldsymbol{\sigma}=(1-D)\bar{\boldsymbol{\sigma}}$ is factored into the degradation damage, (1 – D), and the effective stress, $\bar{\boldsymbol{\sigma}}$:

$$\bar{\boldsymbol{\sigma}} = \mathbf{E}_0 : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p) \in \{\bar{\boldsymbol{\sigma}} \mid F(\bar{\boldsymbol{\sigma}},\boldsymbol{\kappa}) \le 0\} \tag{1}$$

where $\boldsymbol{\varepsilon}^p$ is the plastic strain and $\boldsymbol{\kappa} = [\kappa_t \quad \kappa_c]^{\mathrm{T}}$ is a damage variable vector consisting of two monotonically-increasing scalars: the tensile damage variable κ_t and the compressive damage variable κ_c.The yield function includes two cohesion variables as follows:

$$F(\bar{\boldsymbol{\sigma}},\boldsymbol{\kappa}) = \frac{1}{1-\alpha}\left[\alpha I_1 + \sqrt{3J_2} + \beta(\boldsymbol{\kappa})\langle\hat{\sigma}_{max}\rangle - c_c(\boldsymbol{\kappa})\right] \tag{2}$$

where $\hat{\sigma}_{\max}$ denotes the algebraically maximum principal stress, and is a parameter which is evaluated by the initial shape of the yield function. The evolution of the yield function is determined by the parameter $\beta = \frac{c_c(\boldsymbol{\kappa})}{c_t(\boldsymbol{\kappa})}(1-\alpha) - (1+\alpha)$. Using a multiplicative parameter $0 \le s \le 1$ on the tensile degradation variable D_t , the degradation damage variable in Eq. (1) is defined as: $D = 1 - (1 - D_c(\boldsymbol{\kappa}))(1 - sD_t(\boldsymbol{\kappa}))$, where D_c is the compressive degradation variable.

A reinforced concrete model suggested by Lee (2001) is used, in which the plastic-damage model represents initiation and localization of tensile cracking and compressive crushing damage, as well as stiffness degradation and stiffness recovery on crack closing of concrete. In the reinforced concrete model, the constitutive model for the steel reinforcement (Filippou, Popov *et al.* 1983) is implemented in truss elements to represent reinforcing bars separately from concrete, and these bars are connected to surrounding concrete through imaginary bond-slip material, which is modeled by the discrete link (Eligehausen, Popov *et al.* 1983).

Since the damage variable of the Lee and Fenves plastic-damage model gives the fundamental information for measuring a damage level of the entire structure subject to cyclic and dynamic loading, it can be used to generate a major damage index parameter, the volumetric damage ratio K_v, which is defined as:

$$K_v = \frac{\upsilon \int_{\Omega} \kappa_t d\Omega}{V} \tag{3}$$

where υ is the modification factor, Ω is the volumetric domain, and *V* is the total volume of a column. From the nonlinear regression of experimental column test data, the global damage index, χ is defined in the form as:

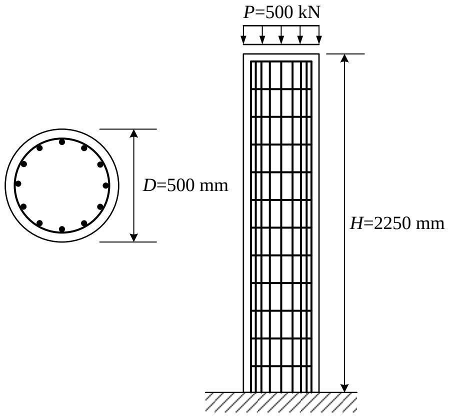
$$\chi = cQ(\rho_w)K_v^{1.5} \tag{4}$$

where *c* is a coefficient, and $Q(\rho_w)$ is a confinement factor function of the lateral confinement reinforcement ratio, ρ_w, in percentage. In contrast to the response-based damage index such as Park and Ang's one (Park and Ang 1985), the suggested global damage index is well-defined in the form of a single monotonically-increasing function of the volume weighted average of local damage distribution, and provides the necessary computability and objectivity because it does not require to compute the ultimate displacement and yield strength.

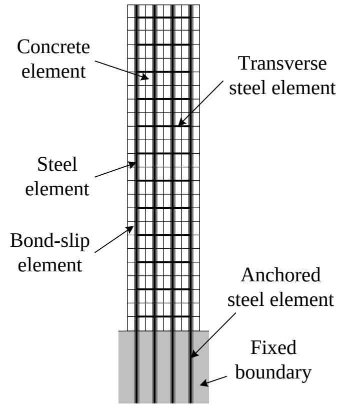
3. Mesh sensitivity analysis of the seismic damage index

3.1 Modeling of a reinforced concrete column

To apply the damage index χ in seismic damage quantification, a reinforced concrete column subjected to a lateral cyclic loading at the top is considered. Fig. 2(a) shows the configuration of the column, of which the diameter and height are 500 mm and 2250 mm, respectively. The column is fixed at the bottom end while supporting a vertical static load of 500 kN. The compressive strength of concrete is 28 MPa. Longitudinal and transverse reinforcement ratios of the column are 1.89% and 0.39%, respectively. Fig. 2(b) shows a finite element model of the reinforced concrete column. The model consists of a plane stress isoparametric quadrilateral elements with the Lee and Fenves plastic damage model for concrete, nonlinear truss element for rebar, and nonlinear bond-slip connection element with zero length. The column is subjected to a lateral displacement cyclic loading at the top as prescribed in Fig. 3(a). By using the described column model, nonlinear finite element analysis was performed to calculate the response of the column under the cyclic loading. Fig. 3(b) represents moment versus drift ratio hysteretic curves obtained by the quasi-static numerical simulation. During the finite element simulation, initiation and localization of concrete tensile cracking were quantified, from which the structural damage index χ could be evaluated using Eq. (4).

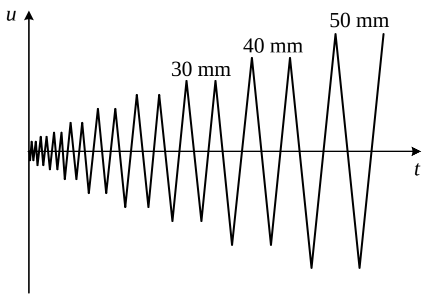


(a) Column configuration

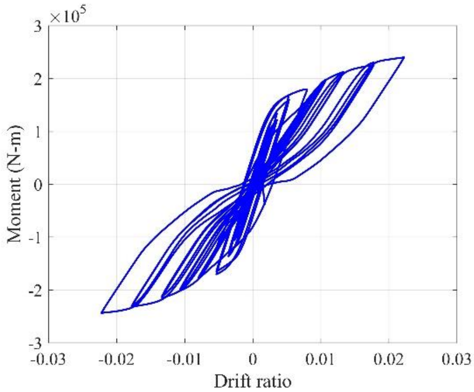


(b) Composition of reinforced concrete structural elements

Fig. 2 Finite element modeling of the reinforced concrete column



(a) Cyclic loading

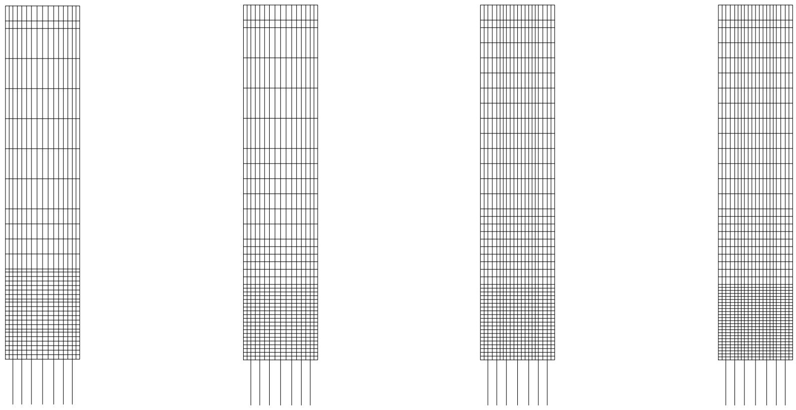


(b) Hysteresis curve

Fig. 3 Moment vs. drift ratio hysteresis curves of the column subjected to a lateral displacement cyclic loading

3.2 Mesh sensitivity of damage index

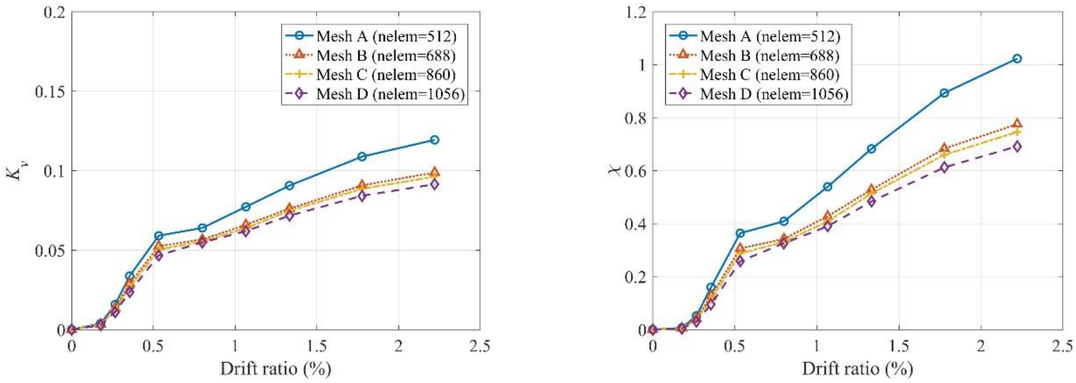
To investigate the mesh sensitivity of the seismic damage index, four different meshes, as shown in Fig. 4, are applied in the finite element analysis of the reinforced concrete column. The meshes have 512, 688, 860, and 1056 concrete elements, respectively. If the number of concrete elements is greater, the number of rebar and bond-slip elements is also greater. Therefore, the column model has increasingly fine mesh from mesh A to mesh D.



(a) Mesh A (b) Mesh B (c) Mesh C (d) Mesh D

Fig. 4 Four meshes used for the modeling of reinforced concrete column; The number of concrete elements is (a) 512, (b) 588, (c) 860, and (d) 1056.

Fig. 5(a) shows the change of volumetric damage ratio K_v with the peak drift ratio of cyclic loading for the four mesh cases. The graphs indicate that volumetric tensile damage over the entire column domain increases monotonically as the peak drift ratio is increased. In Fig. 5(b), the seismic damage index curves are plotted from the finite element simulation results for each of the mesh cases. As the peak drift ratio in the cyclic loading is increased, the damage index values are monotonically increased. It can be seen, from Fig. 5, that the values of K_v and χ converge as the number of elements in the column mesh increases. When using the smaller number of elements, the damage index tends to overestimate damage state for each of the peak drift ratios considered. To overcome such mesh sensitivity problem of the global damage index χ and thus to improve the objectivity of the damage index, a correction method for reflecting mesh density is proposed for the damage index by using nonlinear data regression.



(a) Volumetric damage ratio, K_v (b) Seismic damage index, χ

Fig. 5 Change of K_v and χ with peak drift ratio of the reinforced concrete column subjected to a cyclic loading

4. Damage index reflecting mesh density

Since the seismic damage index χ is a function of K_v as in Eq. (4), it can be modulated to reflect mesh density by considering the mesh dependency of K_v. To establish the relation of K_v with mesh density, the values of K_v at each drift ratio are optimized by regression analysis to the number of elements. Specifically, a nonlinear regression equation of power function form as in Eq. (5) is utilized to set up the relationship at each drift ratio:

$$K_v = a_0 n^{a_1} \tag{5}$$

In Eq. (5), a_0 and a_1 are power function coefficients, and n is the number of elements normalized with the largest number of elements in a mesh set. The nonlinear regression analysis can be performed at each drift ratio to obtain the power function coefficients. The coefficients a_0 and a_1 obtained by the nonlinear regression analysis at each drift ratio are listed in Table 1. Fig. 6 shows the regression curves of K_v for the normalized number of elements at each drift ratio.

Table 1 Coefficients a_0 and a_1 of power function $K_v = a_0 n^{a_1}$

Power function coefficients	Drift ratio (%)								
	0.18	0.27	0.36	0.53	0.80	1.07	1.33	1.78	2.22
a_0	0.0028	0.0111	0.0238	0.0467	0.0535	0.0603	0.0698	0.0820	0.0889
a_1	-0.4473	-0.4749	-0.4762	-0.3152	-0.2178	-0.3119	-0.3260	-0.3584	-0.3723

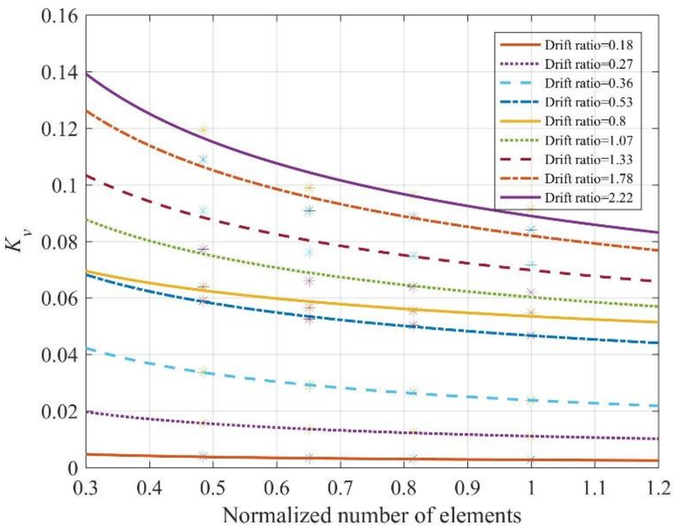
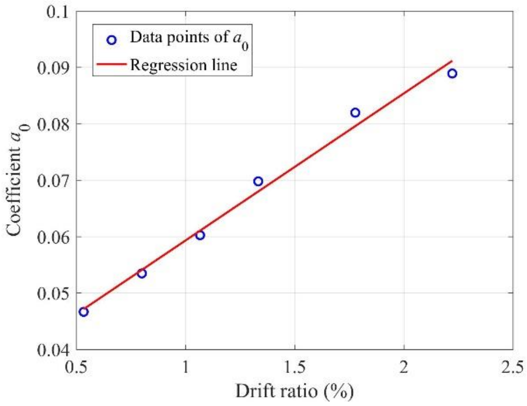
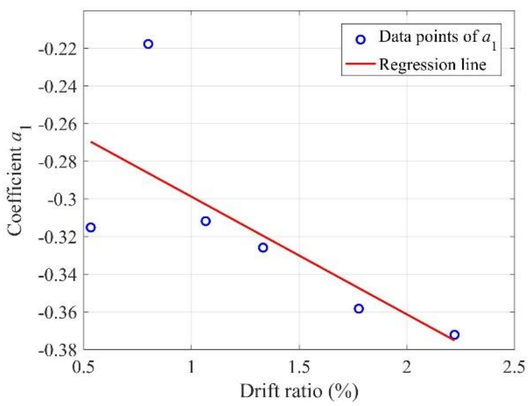


Fig. 6 Regression curves of K_v for normalized number of elements in finite element mesh

To obtain the optimal relationship between the drift ratio and the power function coefficients, a linear regression analysis is performed for each of the coefficients a_0 and a_1. Fig. 7 shows the regression lines for the power function coefficients against drift ratio. From the regression line equations, one can get the coefficients a_0 and a_1 for specific drift ratio so as to calculate K_v by Eq. (5). Then the calculated value of K_v is used in Eq. (4) to evaluate the damage index χ. By the described procedure, mesh density can be reflected in calculating the global damage index χ for structures subjected to cyclic or dynamic loadings. Fig. 8 represents the corrected damage index values against drift ratio for the reinforced concrete column modeled with meshes A to D.

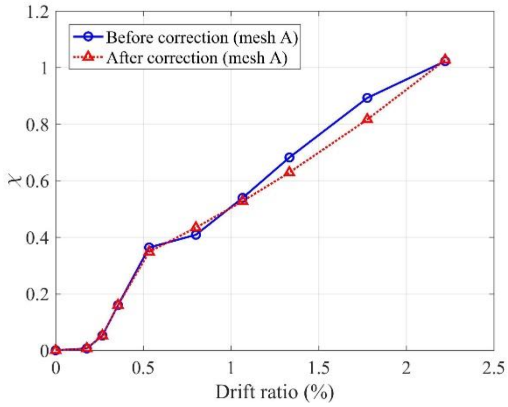


(a) a_0

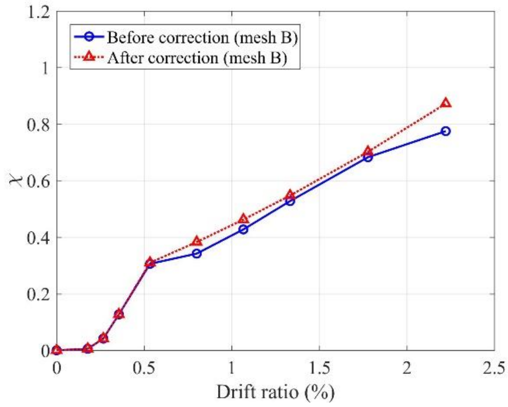


(b) a_1

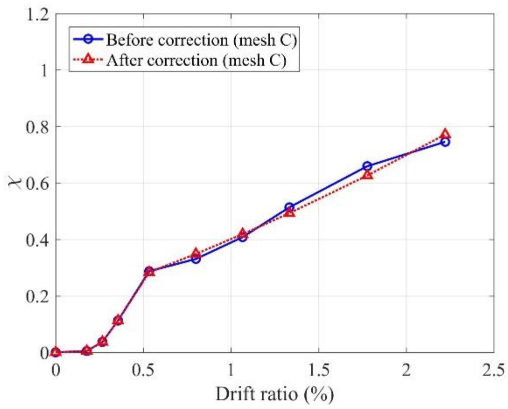
Fig. 7 Regression lines for the power function coefficients a_0 and a_1



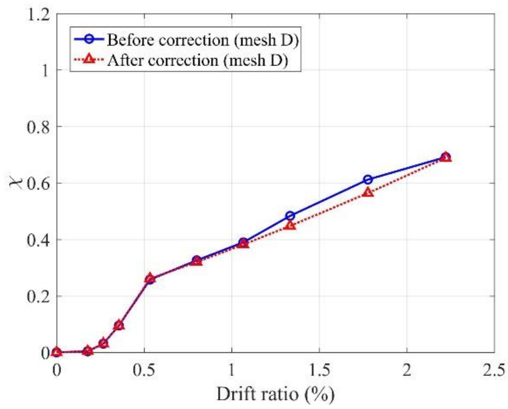
(a) Mesh A



(b) Mesh B



(c) Mesh C



(d) Mesh D

Fig. 8 Damage index χ corrected to reflect mesh density

5. Conclusions

Mesh sensitivity in evaluating a global seismic damage index χ based on a local tensile damage parameter obtained from a nonlinear finite element analysis has been studied. Multiple finite element meshes for a reinforced concrete column were used to investigate the effect of mesh density on the damage index value with a quasi-static cyclic loading condition. According to simulation results, damage states of the test column tends to be overestimated when the mesh is less dense. To overcome the mesh sensitivity problem of the damage index, a correction method is suggested based on nonlinear regression of volumetric tensile damage ratio data. The corrected damage index can ensure the objectivity in evaluating the damage state of structure from the results of nonlinear finite element analysis with various meshes.

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