

## **Efficient decision making for multi-objective probabilistic optimum inspection and monitoring planning of RC structures**

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### **ABSTRACT**

This paper deals with the efficient decision making for multi-objective optimum inspection and monitoring planning of RC structures. The formulations of the objectives for inspection and monitoring planning consider the uncertainties associated with the corrosion initiation and propagation, damage detection, and the effect of maintenance on the service life and cost. The objective reduction approach is used to identify the essential objectives. The multiple attribute decision making is applied to estimate the weights of the essential objectives and select a well-balanced solution among the Pareto solutions for inspection and monitoring planning of RC structures.

### **1. INTRODUCTION**

Corrosion is one of the most critical deterioration mechanisms occurring in RC structures (NCHRP 2006). In general, corrosion damage cannot be detected perfectly, and the effects of inspection and maintenance on the service life and life-cycle cost of a deteriorating RC structure are uncertain. For this reason, probabilistic approaches for establishing inspection and monitoring plans need to be developed (Frangopol 2011, 2016). This paper presents the six objectives for the optimum inspection planning and five objectives for the optimum monitoring plans. These multiple objectives are considered simultaneously through the multi-objective optimization process. The multiple attribute decision making (MADM) is applied to estimate the weights of the essential objectives and select a well-balanced solution among the Pareto solutions for inspection and monitoring planning of RC structures (see Fig. 1). The essential objectives are identified by using the dominance relation-based objective reduction approach.

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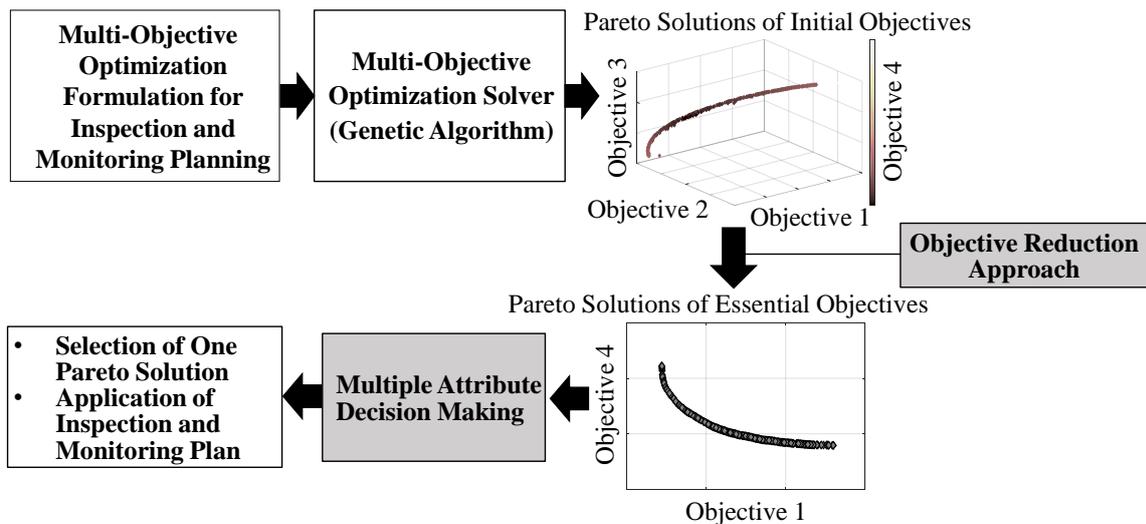


Fig. 1 Decision making process for optimum inspection and monitoring planning.

## 2. OBJECTIVE FUNCTIONS FOR INSPECTION AND MONITORING PLANNING

The objective functions for optimum inspection and monitoring planning are formulated considering the uncertainties associated with damage initiation and propagation, damage detection, and the effect of maintenance on the service life and life-cycle cost. The formulations of the objectives for optimum inspection planning are provided in [Table 1](#), where  $N_{ins}$  = number of inspections;  $t_{ins,i}$  =  $i$ th inspection time;  $t_{life,i}$  = service life of the structure after the  $i$ th inspection;  $P_{ins,i}$  = probability of damage detection of the  $i$ th inspection;  $f_T(t)$  = PDF of the damage occurrence time;  $P_s$  = reliability;  $N_{mnt}$  = number of available maintenance types;  $d$  = degree of corrosion damage for the  $i$ th inspection;  $d_{ma,j}$  = critical degree of corrosion damage for the  $j$ th maintenance action.  $C_{ins}$ ,  $C_{ma}$  and  $C_{fail}$  are costs of inspection, maintenance and failure, respectively. Furthermore, the multi-objective optimum monitoring planning is investigated based on the five objectives:  $O_{m,1}$  = minimizing the expected damage detection delay  $E(t_{dm})$ ;  $O_{m,2}$  = minimizing the expected maintenance delay  $E(t_{mn})$ ;  $O_{m,3}$  = maximizing reliability index  $\beta$ ;  $O_{m,4}$  = maximizing the expected service life extension  $E(t_{ex})$ ;  $O_{m,5}$  = minimizing the expected life-cycle cost  $C_{lcc}$ . These five objectives can be formulated based on the objectives for optimum inspection planning in [Table 1](#).

## 3. OBJECTIVE REDUCTION APPROACH

The initial objective set of the multi-objective optimization problem consists of the essential and redundant objectives. The Pareto front of the multi-objective optimization problem is affected by only the essential objectives. The redundant objectives can be ignored, because there is no effect of these objectives on the Pareto front. By using the dominance relation-based objective reduction approach developed by [Brockhoff \(2006, 2009\)](#), the essential and redundant objectives can be identified. In this approach, the

Pareto optimal solution set is represented by the dominance relations among the objective values. As a result, this approach provides the normalized degree of conflict  $\Delta_{norm}$  between the initial objective set  $\Lambda_I$  and reduced objective set  $\Lambda_R$ . If  $\Delta_{norm}$  is equal to zero, the Pareto front of  $\Lambda_I$  is the same as that of  $\Lambda_R$ .

**Table. 1** Formulations of the objectives for inspection planning.

$O_{1,1}$	Maximizing the probability of damage detection $P_{dt}$ $P_{dt} = \sum_{i=1}^{N_{ins}} \left[ \prod_{j=1}^i \{ P(t_{ins,j} \leq t_{life,o}) \cdot (1 - P_{ins,j-1}) \} \cdot P_{ins,i} \right]$
$O_{1,2}$	Minimizing the expected damage detection delay $E(t_{dm})$ $E(t_{dm}) = \sum_{i=1}^{N_{ins}+1} \left[ \int_{t_{ins,i-1}}^{t_{ins,i}} \{ t_{dm,i} \cdot f_T(t) \} dt \right]$
$O_{1,3}$	Minimizing the expected maintenance delay $E(t_{mn})$ $E(t_{mn}) = \sum_{i=1}^{N_{ins}+1} \left[ \int_{t_{ins,i-1}}^{t_{ins,i}} \{ t_{mn,i} \cdot f_T(t) \} dt \right]$
$O_{1,4}$	Reliability index $\beta$ $\beta = \Phi^{-1}(P_s)$
$O_{1,5}$	Expected service life extension $E(t_{ex})$ $E(t_{ex}) = E \left[ \sum_{i=1}^{N_{ins}} \sum_{j=1}^{N_{mt}} P(t_{ins,i} \leq t_{life,j-1}) \cdot P(d_{ma,j} \leq d_i < d_{ma,j+1}) \cdot t_{ex,j}^* \right]$
$O_{1,6}$	Expected life-cycle cost $C_{lcc}$ $C_{lcc} = C_{ins} + C_{ma} + C_{fail}$

#### 4. MULTIPLE ATTRIBUTE DECISION MAKING

All the Pareto optimal solutions of the multi-objective optimization problem can be applied to inspection and monitoring planning. Since practical structural management may require only one inspection and monitoring plan, a well-balanced solution among the Pareto optimal solutions needs to be selected. MADM can be used to select the best Pareto optimal solution with the largest overall assessment value. The overall assessment value  $OV_i$  of  $i$ th Pareto optimal solution is computed as (Yoon 1995)

$$OV_i = \sum_{j=1}^{N_{obj}} w_j f_{ij} \quad (1)$$

where  $N_{obj}$  = number of objectives of the multi-objective optimization;  $f_{ij}$  =  $j$ th normalized objective value associated with the  $i$ th Pareto solution. It should be noted that the weight of the  $j$ th objective  $w_j$  is equal or larger than zero, and  $\sum w_j$  is one. In this paper, the weight of the objective is determined using standard deviation (SD), criteria importance through inter-criteria correlation (CRITIC), correlation coefficient and standard deviation (CCSD) methods. The detailed information on these methods is available in [Diakoulaki \(1995\)](#) and [Wang \(2010\)](#).

## 5. APPLICATION

In this study, the multi-objective optimization for inspection and monitoring planning is applied to the I-39 Northbound Bridge in Wisconsin, USA. The top transverse position of reinforcement bars of the deck is considered as a critical location subjected to corrosion. The deterministic and probabilistic variables for the corrosion propagation and cost estimation of this application can be found in [Kim \(2011, 2017\)](#). The multi-objective optimization problem is formulated as

$$\text{Find } \mathbf{t}_{ins} = \{t_{ins,1}, t_{ins,2}, \dots, t_{ins,N_{ins}}\} \quad (2a)$$

$$\text{for } \Lambda_1 = \{O_{1,1}, O_{1,2}, O_{1,3}, O_{1,4}, O_{1,5}, O_{1,6}\} \quad (2b)$$

$$\text{such that } 1 \text{ year} < t_{ins,i} - t_{ins,i-1} < 20 \text{ years} \quad (2c)$$

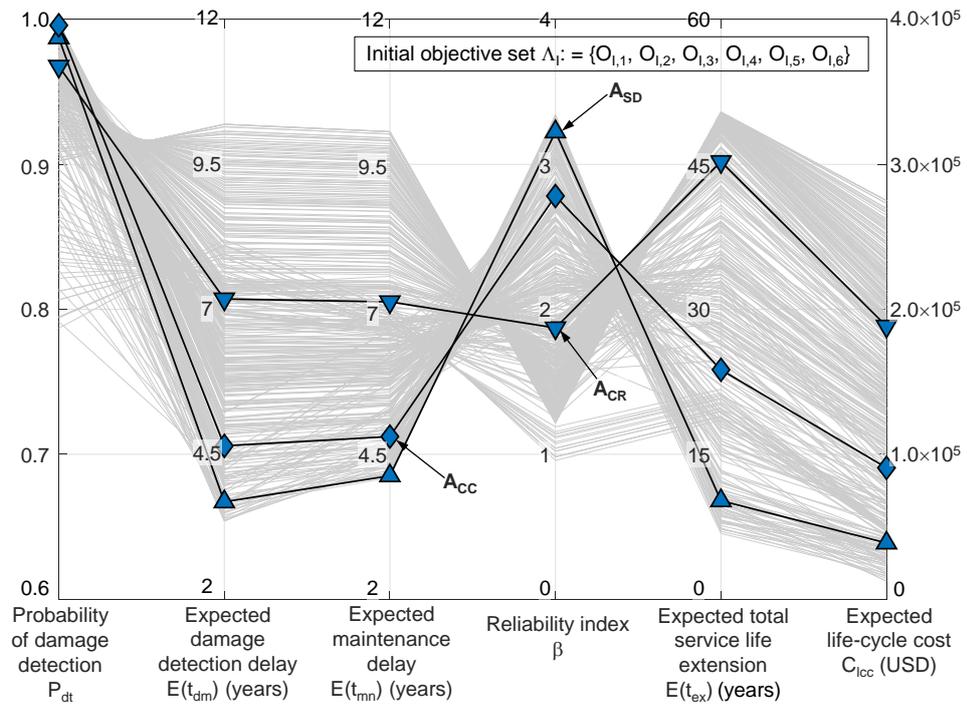
$$\text{given } N_{ins} \quad (2d)$$

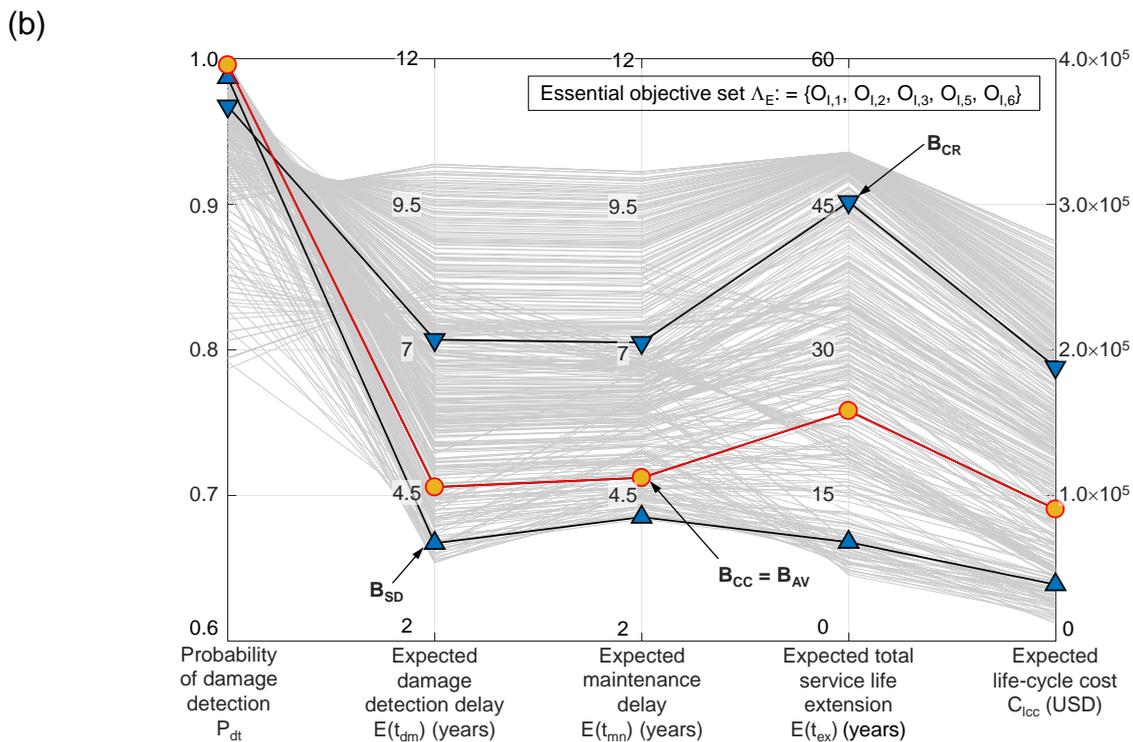
The design variables are inspection times  $\mathbf{t}_{ins}$  (see [Eq. \(2a\)](#)), the time interval between inspections should be larger than 1 year and less than 20 years (see [Eq. \(2c\)](#)), and the number of inspection  $N_{ins}$  is given (see [Eq. \(2d\)](#)). The objectives  $O_{1,1}$  to  $O_{1,6}$  in [Eq. \(2b\)](#) are defined in [Table 1](#). The multi-objective optimization problem defined in [Eq. \(2\)](#) is solved using the genetic algorithms provided in MATLAB R2016b.

As shown in [Fig. 2](#), the Pareto solutions are illustrated in the parallel coordinate system, when the number of inspections  $N_{ins} = 3$  is considered. The six vertical axes represent the objective values associated with  $O_{1,1}$  to  $O_{1,6}$ . By using the SD, CRITIC, and CCSD methods, the solutions  $A_{SD}$ ,  $A_{CR}$  and  $A_{CC}$  in [Fig. 2\(a\)](#) are selected. The inspection plans for the solutions  $A_{SD}$ ,  $A_{CR}$  and  $A_{CC}$  are shown in [Fig. 3](#). Furthermore, the objective reduction approach is used to identify the essential and redundant objectives. As a result,  $O_{1,4}$  = maximizing the reliability index  $\beta$  is the redundant objective, since  $\Delta_{norm}$  between the initial objective set  $\Lambda_1$  and the essential objective set  $\Lambda_E = \{O_{1,1}, O_{1,2}, O_{1,3}, O_{1,5}, O_{1,6}\}$  is equal to zero. Considering the essential objective set  $\Lambda_E$ , the solutions  $B_{SD}$ ,  $B_{CR}$  and  $B_{CC}$  are selected as shown in [Fig. 2\(b\)](#). In order to consider the weights computed by using the SD CRITIC and CCSD methods simultaneously, the average of  $w_{j,SD}$ ,  $w_{j,CR}$  and  $w_{j,CS}$  is used, where  $w_{j,SD}$ ,  $w_{j,CR}$  and  $w_{j,CS}$  are the weights of the  $j$ th objective by using the SD CRITIC and CCSD methods,

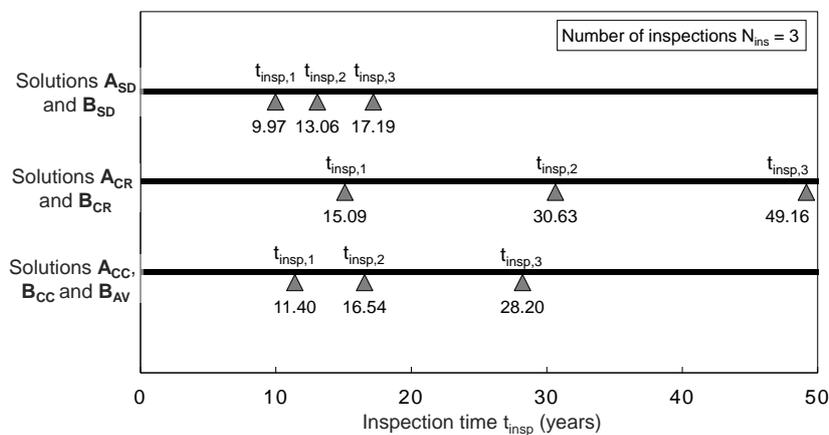
respectively. Figs. 2(b) and 3 shows that the solution  $B_{AV}$  based on the average weight is the same as the solutions  $A_{CC}$  and  $B_{CC}$ . The inspection plan for the solutions  $A_{CC}$ ,  $B_{CC}$  and  $B_{AV}$  requires three inspections applied at 11.40, 16.54 and 28.20 years (see Fig. 3). This inspection plan results in  $P_{dt} = 0.996$ ,  $E(t_{dm}) = 4.64$  years,  $E(t_{mn}) = 4.81$  years,  $E(t_{ex}) = 23.75$  years and  $C_{lcc} = 90652$  (USD) (see Fig. 2(b)).

(a)





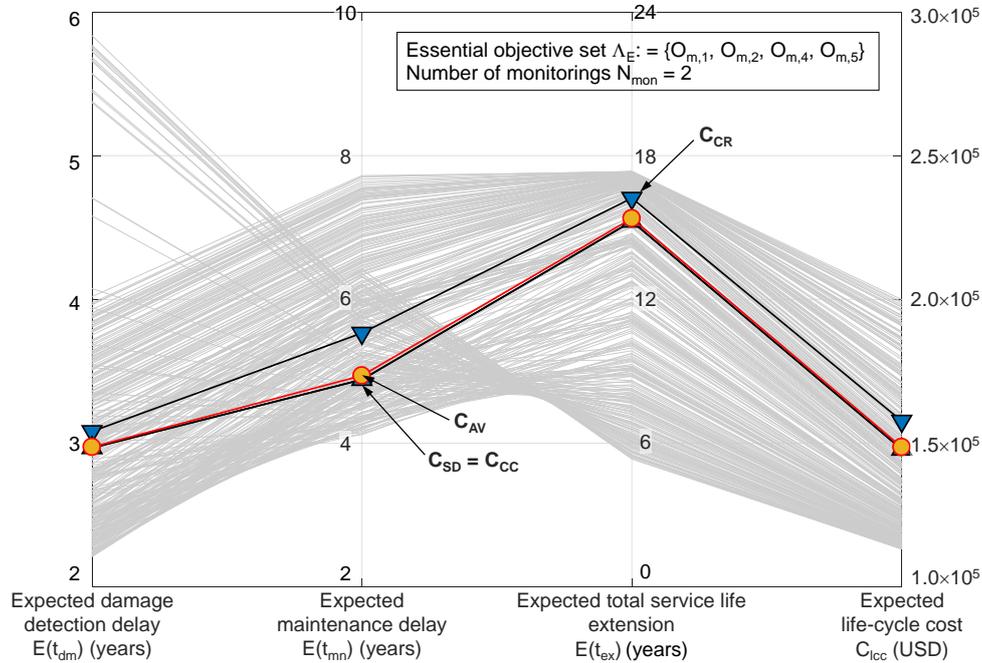
**Fig. 2** Pareto solution set for optimum inspection planning with  $N_{ins} = 3$ : (a) initial objective set  $\Lambda_I$ ; (b) essential objective set  $\Lambda_E$ .



**Fig. 3** Optimum inspection plans associated with the number of inspections  $N_{ins} = 3$  considering the essential objective set  $\Lambda_E$ .

In a similar way, the optimum monitoring plans can be established considering the five objectives:  $O_{m,1}$  = minimizing the expected damage detection delay  $E(t_{dm})$ ;  $O_{m,2}$  = minimizing the expected maintenance delay  $E(t_{mn})$ ;  $O_{m,3}$  = maximizing reliability index  $\beta$ ;  $O_{m,4}$  = maximizing the expected service life extension;  $O_{m,5}$  = minimizing the expected life-cycle cost  $C_{lcc}$ . **Fig. 4** shows the Pareto solution set of the essential

objective set  $\Lambda_E = \{O_{m,1}, O_{m,2}, O_{m,4}, O_{m,5}\}$  for the number of monitoring  $N_{\text{mon}} = 2$ . Also, the selected solutions  $C_{SD}$ ,  $C_{CR}$ ,  $C_{CC}$  and  $C_{AV}$  based on the SD CRITIC and CCSD methods and average weights of the objectives are indicated in Fig. 4.



**Fig. 4** Pareto solution set for optimum monitoring planning considering the essential objective set  $\Lambda_E$  for  $N_{\text{mon}} = 2$ .

## 5. CONCLUSIONS

This study deals with the multi-objective optimization process for optimum inspection and monitoring planning of deteriorating RC structures. A well-balanced solution of Pareto solution set is selected by using MADM with the essential objectives. The dominance relation-based objective reduction approach results in identifying the essential and redundant objectives. The accuracy of the inspection and monitoring planning presented in this paper is affected by the models for (a) the corrosion initiation and propagation, (b) the relation between degree of corrosion damage and probability of damage detection, and (c) the service life extension and cost by applying the maintenance actions. The information obtained from each inspection and monitoring can be used to improve the accuracy of the existing inspection and monitoring schedule through the updating process.

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