

## **Effect of nonlinear hydrostatic stiffness on response of floating wind turbine**

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### **ABSTRACT**

The booming demand for wind energy has pushed the offshore wind turbine into much deeper water. At water depth greater than 60 m the floating wind turbines are considered to be the most economical and feasible alternative to that of the fixed supported offshore wind turbines. The stability of floating platform is one of the major challenges and is primarily achieved through mooring lines and hydrostatic buoyancy of the platform. Based on the assumption of the small displacement of the platform, in most of the studies, the hydrostatic stiffness is considered to vary linearly with the platform position. The present study is focused on the effect of nonlinear hydrostatic stiffness on the response of floating wind turbine. The floating wind turbine is modeled as a multi-body system and the equation of motion is obtained from the principle of conservation of momentum. The effect of hydrostatic nonlinearity is demonstrated through the analysis of OC3-Hywind floating wind turbine model. In the case of small motion, the response for the linear and nonlinear cases is identical. However, for large displacement, the effect of hydrostatic nonlinearity was found to be more pronounced.

### **1. INTRODUCTION**

The advancement of maritime industry has moved the installation of wind turbine further from shore and in deeper water (Arapogianni 2013). At higher water depth (> 60 m) the fixed supported wind turbine lose its economic advantage and are replaced by floating wind turbine. The floating wind turbine is in its developing stage and various challenges have to be overcome to enable the development of the floating wind turbine industry. Floating wind turbine is a complex multibody system, subjected to various types of environmental forces and is restrained by mooring lines. In the last decades, various studies have been conducted by researchers on the response of floating wind turbine focusing on various types of loading i.e. the wind, waves and mooring lines, acting on the floating wind turbines (Gutiérrez-Romero 2016; Li 2012). Despite the fact, that there are various studies in the literature on the response of floating wind turbine, nonlinear hydrostatics of floating platform has relatively received less attention and

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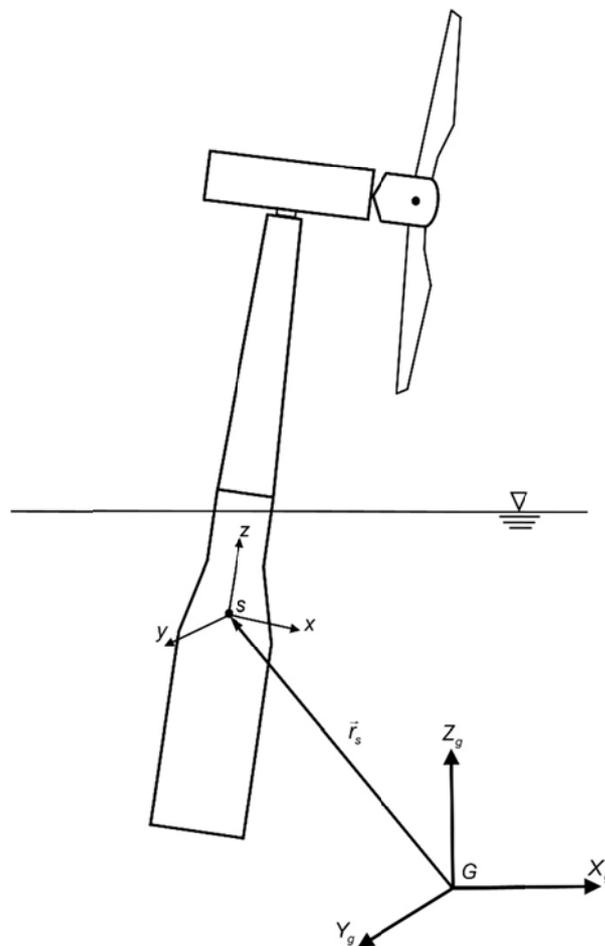
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need to be studied further. In this study, we have investigated the effect of hydrostatic nonlinearity on the response of floating wind turbine.

## 2. FORMULATION

The offshore Floating wind turbine is a multibody system consisting of floating platform, tower, nacelle, and rotor. Various analytical methods are available for the formulation of multibody system. In this study, the equation of motion of the floating wind turbine is obtained through the use of Newton-Euler dynamical principal (Wang 2013). The floating wind turbine is modeled as three rigid bodies supporting substructure (platform and tower), nacelle and rotor, connected together through mechanical joints, as shown in Fig. 1. The multi-body system has a total of 6 degrees of freedom with additional prescribed degree of freedom of nacelle yaw and rotor spin. The global reference frame ( $G, X_g, Y_g, Z_g$ ) and system body reference frame ( $s, x, y, z$ ) are used to describe the position and orientation floating wind turbine in inertial space. The position of a floating wind turbine in the global reference frame is denoted by



$\vec{r}_s = [x_s, y_s, z_s]^T$ , and the orientation of local reference frame with respect to the global reference frame is defined by Euler angles  $\vec{\Phi} = [\phi_1, \phi_2, \phi_3]^T$ . The translational and rotational equation of motion of the whole system can be obtained using Newton-Euler approach, as follows:

$$\sum \vec{F} = m_s \ddot{\vec{r}}_s \quad (1a)$$

$$\sum \vec{M} = \frac{d\vec{H}_s}{dt} + \vec{\omega}_s \times \vec{H}_s \quad (1b)$$

where  $\sum \vec{F}$  and  $\sum \vec{M}$  represent applied forces and moments, respectively.  $m_s$  is the mass of the whole system consists of a platform, tower, nacelle, and rotor;  $\vec{r}_s$  is the position vector of center or mass of the whole system in the inertial reference frame and the superposed dot show rate of change with respect to time;  $\vec{H}_s$  and  $\vec{\omega}_s$  represents the angular momentum and angular velocity of the system, respectively. The forces  $\sum \vec{F}$  and moments  $\sum \vec{M}$  mainly consist of:

$$\sum \vec{F} = \vec{F}_w + \vec{F}_T + \vec{F}_g + \vec{F}_b + \vec{F}_m \quad (2a)$$

$$\sum \vec{M} = \vec{M}_w + \vec{M}_T + \vec{M}_b + \vec{M}_m \quad (2b)$$

where  $\vec{F}_w$ ,  $\vec{F}_T$ ,  $\vec{F}_g$ ,  $\vec{F}_b$  and  $\vec{F}_m$  represent forces due to waves, wind, gravity, buoyancy and mooring lines, respectively. Similarly  $\vec{M}_w$ ,  $\vec{M}_T$ ,  $\vec{M}_b$  and  $\vec{M}_m$  are the moments due to waves, wind, buoyancy and mooring lines, respectively. The hydrodynamic wave forces and moments are determined using Morison's equations, as follows:

$$\vec{F}_w = \int_{h_s} \left( \frac{C_m \rho \pi D^2}{4} \dot{\vec{V}}_{fn} + \frac{C_D \rho D^2}{2} \vec{V}_m |\vec{V}_m| - \frac{C_a \rho \pi D^2}{4} \dot{\vec{V}}_{sn} \right) dz \quad (3a)$$

$$\vec{M}_w = \int_{h_s} \left( \vec{r}_z \times T^T \left( \frac{C_m \rho \pi D^2}{4} \dot{\vec{V}}_{fn} + \frac{C_D \rho D^2}{2} \vec{V}_m |\vec{V}_m| - \frac{C_a \rho \pi D^2}{4} \dot{\vec{V}}_{sn} \right) \right) dz \quad (3b)$$

where  $\rho$  is water density;  $D$  is the diameter of the structure;  $h_s$  is the submerged height of the platform;  $C_d$ ,  $C_m$  and  $C_a$  are drag, inertia and added mass coefficients, respectively;  $\vec{V}_m$  is relative velocity vector normal to the axis of the floating platform;  $\dot{\vec{V}}_{fn}$  and  $\dot{\vec{V}}_{sn}$  denotes the acceleration of fluid particle's and structure normal to axis of the

member, respectively; and  $T$  is the transformation matrix from the local reference frame to the global reference frame. The transformation matrix  $T$  is obtained from the Euler angles in the sequence of 1-2-3 rotation. Mooring lines are modeled quasi-statically using catenary cable element (Jonkman 2009). The wind thrust on the total swept area is determined as follows (Knauer 2006):

$$\vec{F}_T = \frac{1}{2} \rho_a A u_{rel}^2 C_t \quad (4a)$$

where  $\rho_a$  is air density;  $A$  is rotor swept area;  $u_{rel}$  is relative wind velocity with respect to the rotor, and  $C_t$  is thrust coefficient. If  $\vec{r}_r$  is the position vector of the rotor from system body reference frame, the moment due to thrust force can be obtained as follows:

$$\vec{M}_T = \vec{r}_r \times \vec{F}_T \quad (4b)$$

The hydrostatic forces  $\vec{F}_b$  and moments  $\vec{M}_b$  can be obtained from the hydrostatic stiffness of the floating platform which depends on the submerged volume and center of buoyancy of the floating platform, which changes with the displacement and rotation of the platform. Linear hydrostatic stiffness  $K_L$  is determined from the initial position of the floating platform, assuming that the rotation of the platform is small and its effect can be neglected. If hydrostatic force and moment are denoted by a single vector as  $\vec{F}_B = [\vec{F}_b, \vec{M}_b]^T$  and displacement vector is denoted as  $\vec{X}_d = [\vec{r}_s, \vec{\Phi}]^T$ , the  $\vec{F}_B$  can be determined based on linear hydrostatic stiffness as  $\vec{F}_B = K_L \vec{X}_d$ . The nonlinear hydrostatic stiffness  $K_{NL}$  is determined at each time step by determining the actual submerged volume and center of buoyancy in the displaced position of the platform, thus taking into account the effect translation and rotational displacements. The  $\vec{F}_B$  can be obtained from nonlinear hydrostatic stiffness as  $\vec{F}_B = K_{NL} \vec{X}_d$ .

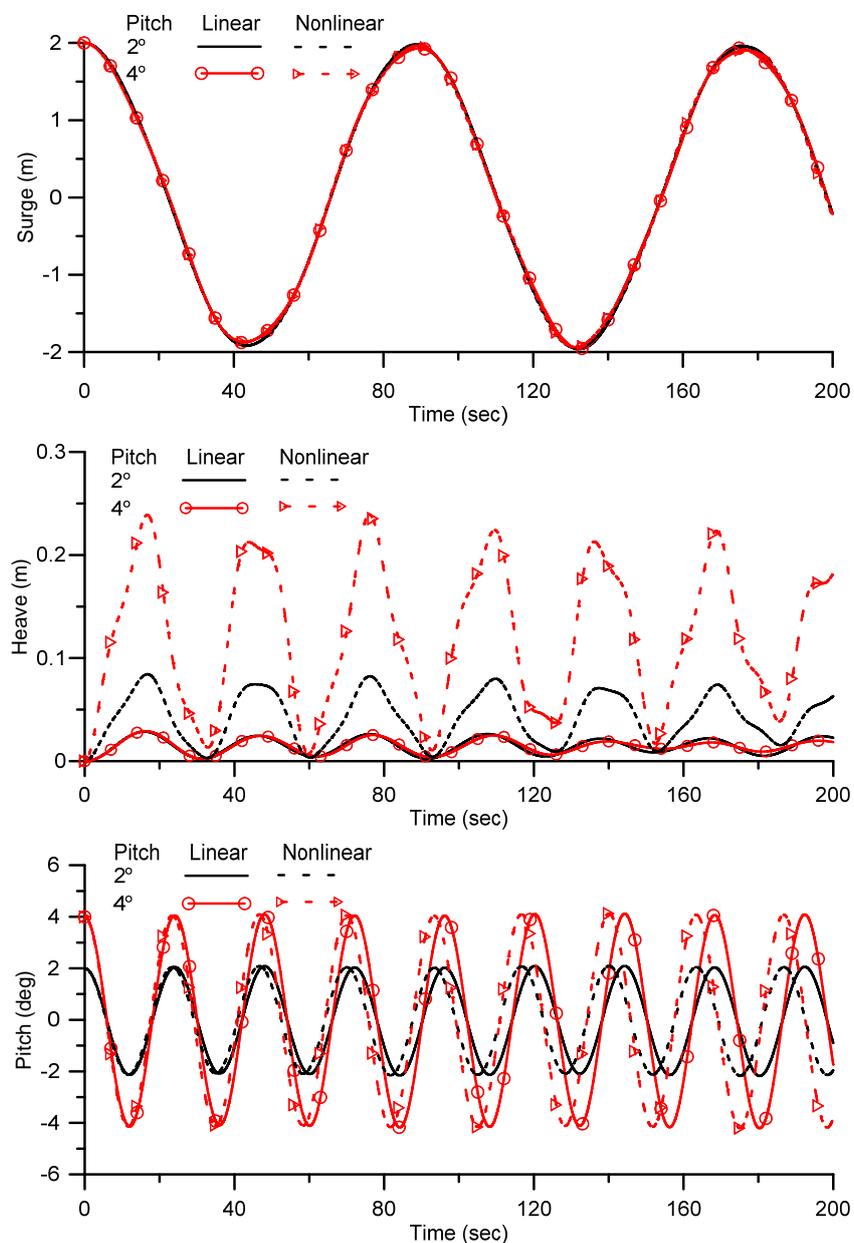
### 3. RESULTS AND DISCUSSION

To demonstrate the effect of nonlinear hydrostatic stiffness on the response of floating wind turbine, OC3-Hywind model (Jonkman 2010) is subjected to initial surge displacement of 2 m and pitch rotation of 2, and 4 degrees. The detail properties of OC3-Hywind model and mooring system can be found in Jonkman (2010). The hydrodynamic and aerodynamic forces are assumed zeros. The surge, heave and pitch response of floating wind turbine is compared for both the linear and nonlinear hydrostatic stiffness and is plotted in Fig. 2. The figure shows that surge response is identical for both the cases and the effect of hydrostatic nonlinearity is negligible in surge direction. The heave displacement shows a significant influence of hydrostatic

nonlinearity and the effect of hydrostatic nonlinearity increases with increase in the pitch rotation. **Figure 2** shows that the effect hydrostatic nonlinear results in a lag in the pitch rotation and it increases with increase in the rotation.

#### 4. CONCLUSIONS

The study focused on the effect of hydrostatic nonlinearity on the response of offshore floating wind turbine. The floating wind turbine is modeled using Newton-Euler approach. The influence of hydrostatic nonlinearity on the response floating wind turbine is demonstrated through analysis of OC3-Hywind model. The study shows that for the small displacement the effect of hydrostatic nonlinearity is negligible. However, in the case of large displacement, the effect of hydrostatic nonlinearity profoundly increases in heave, pitch, and roll direction.



**Fig. 2** Effect of Hydrostatic nonlinearity on the surge, heave and pitch responses of floating wind turbine

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