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Neutral-axis Identification using strain and acceleration measurements

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ABSTRACT

Structural health monitoring aims to continuously monitor and evaluate structural performance by measuring structural parameters. Among many parameters, the change in the location of the neutral axis within a cross-section can indicate unusual structural behaviors such as reduction in stiffness. This paper proposes a methodology to estimate the neutral axis of a beam structure using acceleration and strain measurement. The numerical simulation was conducted on simply supported steel-beam model to validate relationship between the magnitude of damage versus change in neutral axis. This paper concludes that the position of the neutral axis can be evaluated using strain and acceleration measurement.

1 INTRODUCTION

The neutral axis is a set of points in the cross-section of a beam at which normal stress and strain vanishes under given loads. The centroid of stiffness is a universal parameter of cross-sections of every load-carrying beam structure. A change in the position of the neutral-axis within a given cross-section indicates unusual structural behaviors. The neutral axis of a beam can be changed not only by the reduction in cross section, but also by unexpected changes in axial force in the case of pre-stressed concrete or temperature. Consequently, the position of the neutral axis can indicate any

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abnormal change in a beam.

A change in location of neutral-axis with respect to structural condition has been shown to be correlated experimentally by several researchers. The neutral-axis can be estimated either by using strain-derived method and data fusion method that uses both strain and acceleration.

The strain-derived method uses as few as two strain values on in the cross-section of a structure with the assumption of linear theory to determine the position of the neutral-axis. Chung, 2008 conducted 4-point flexure test on a 25m PSC girder and experimentally showed the decrease in neutral-axis as at the girder's limit state. Sigurdardottir (2013) experimentally invested the use of neutral axis as a damage indicator for beam-like structure through long-term monitoring of a full-scale bridge. They showed that short-term evaluation of the neutral-axis proved effective in detecting loss of stiffness, but long-term observation was prohibited due to rheological effects in the concrete and nonlinear temperature variations. The of neutral-axis position estimation directly using the measured strain data may also be significantly distorted in the presence of measurement noise and varying load patterns. Xia (2012) and Soman (2016) proposed the use of Kalman Filtering to reduce the signal noise to accurately estimate the position of the neutral-axis for structural health monitoring.

Another approach for neutral-axis positioning is to use both strain and acceleration measurement. Rauert (2011) proposed the data fusion for determining the neutral-axis of a beam structure using a number of acceleration, strain measurement, and FE model. However, the proposed method requires fairly accurate strain and displacement mode shapes as well as a number of sensors. Also they did not investigate either the possibility of using obtained neutral-axis as a damage indicator nor the effect of non-linearity in the model.

This paper proposes data fusion using small number of strain and acceleration sensors to accurately estimate the neutral-axis. Also, the proposed method is able to estimate accurate neural-axis even when subjected to temperature gradient or signal noise. The formulation of the proposed method is presented and numerical simulation on a simple beam model to validate the proposed method is followed. The proposed method is then validated on a reinforced concrete beam that consider nonlinearity in the material.

2 BACKGROUND

2.1 Neutral axis obtained by strain measurements for beam-like structures.

This study focuses on beam-like structures such as bridges. The strain and displacement in modal coordinate can be conveniently expressed in the form of (1).

$$u_i(t) = \sum_j \phi_{ij} q_j(t)$$
$$\varepsilon_k(t) = -u''(x_k, t)y = -y\sum_j \phi_{kj}'' q_j(t) \tag{1}$$

where u_i and ε_k indicate the displacement estimated at *i*-th position, and strain measured at *k*-th position on a structure; y is the neutral axis. A linear relationship between the displacement and strain responses can be formulated using the modal approach. Let us consider a system with displacement and strain responses $\{u\}_{m\times1}$ and $\{\varepsilon\}_{n\times1}$ where *m* and *n* are the numbers of measurements. The displacement and strain measurements can be approximated using the linear combination of the finite number of modes.

$$\{u\}_{m\times1} = \Phi_{m\times r}\{q\}_{r\times1} \tag{2}$$

$$\{\varepsilon\}_{n\times1} = \frac{1}{\alpha}\Phi''_{n\times r}\{q\}_{r\times1} \tag{3}$$

where $\{\Phi\}_{m\times r}$ and $\{\Phi''\}_{n\times r}$ are respective displacement and second-derivative of displacement mode shape matrices, α is the neutral axis, $\{q\}_{r\times1}$ is the modal coordinate, and *r* is the number of used modes. When $n \geq r$, the modal coordinate $\{q\}$ can be obtained from Eq.(4) as

$$\{q\} = \Psi^{+}\{\varepsilon\} \tag{4}$$

where the superscript + denotes the Moore-Penrose pseudo inverse. The strain-displacement relationship is obtained by substituting $\{q\}_{r\times1}$ in Eq.(2) with Eq. (4).

$$\{u\} = \Phi\ \Psi^{+}\{\varepsilon\} \tag{5}$$

Given the strain measurements, displacement responses can be obtained if the displacement and strain mode shapes are known.

$$\{u\}_{m\times1} = D_{m\times n}\{\varepsilon\}_{n\times1} \tag{6}$$

where $D_{m\times n}$ is a transformation matrix. Shin (2012) proposed a displacement-strain relationship for a simply supported beam using sinusoidal displacement mode shapes.

2.2 Peak-power matching algorithm based on strain and acceleration

Peak-power matching algorithm for neutral-axis was proposed by Park (2013). The strain-based displacement at position x_i on a simply-supported beam of length L can be expressed as in Eq.(7) assuming only the first mode and sinusoidal mode shapes.

$$\begin{aligned} u_s(x_i,t) &= \Phi\ \Phi''^{+}\{\varepsilon\} \\ u_s(x_i,t) &= -\frac{1}{\alpha}\Phi\ \Phi''^{+}\,\Phi'' q_1(t) \\ u_s(x_i,t) &= -\frac{1}{\alpha}\sin\left(\frac{\pi}{L}x_i\right)\cos(2\pi f_1 t) \end{aligned} \tag{7}$$

Likewise, displacement also can be estimated at *xi* using acceleration response as in Eq.(8) under the same assumptions.

$$u_{acc}(x_i,t)=\phi(x_i,t)q_1(t)$$
$$acc(x_i,t)=\phi(x_i,t)\ddot{q}_1(t) \tag{8}$$
$$u_{acc}(x_i,t)=\sin(\frac{\pi}{L}x_i)\cos(2\pi f_1 t)=\frac{1}{-(2\pi f_1)^2}acc(x_i,t)$$

The combination of the strain- and acceleration-based methods can provide the information about the neutral axis which can be obtained by adjusting the displacement from strain measurements to the displacement estimated from acceleration. The adjustment can be made in frequency domain by matching the power spectral density of the displacement from strain to that from acceleration as in Eq.(9).

$$\alpha_{x_i}=\sqrt{\frac{S_{x_i}^{disp,acc}(f_1)}{S_{x_i}^{disp,strain}(f_1)}} \tag{9}$$

where α is the scaling factor, $S_{d,acc}$ and $S_{d,strain}$ are the respective power spectral densities of the displacements estimated from acceleration and strain, and f_1 is the first natural frequency of the estimated displacements from acceleration and strain.

2.3 RANSAC

Random Sample Consensus (RANSAC) is a general parameter estimation method that deal with regression problem with large portion of outliers in the given sample data. RANSAC is a resampling technique that generates candidate solutions by using the minimum number observations (data points) required to estimate the underlying model parameters. RANSAC uses the smallest set possible and proceeds to enlarge this set with consistent data points. The steps for implementing the RANSAC algorithm is summarized as follows:

1. Select randomly the minimum number of points required to determine the model parameters.
2. Solve for the parameters of the model $f(x)$
3. Calculate residual r and standard deviation denoted as S of the samples using

$$r_i=y_i-f(x_i,\beta) \tag{10}$$
$$S=\sum_{i=1}^{n}r_i^2 \tag{11}$$

4. Determine how many points from the set of all points fit
5. If the fraction of the number of inliers over the total number points in the set exceeds a predefined threshold τ, re-estimate the model parameters using all the identified inliers and terminate.
6. Otherwise, repeat steps 1 through 4 (maximum of N times).

3 PROPOSED DATA FUSION FOR NEUTRAL-AXIS ESTIMATION

While the neutral-axis estimation method by peak power matching is implemented in frequency domain, the proposed estimation method solve in the time domain in three steps using acceleration and strain-based estimated displacement.

The first step is to apply band-pass filter to extract only the first mode from both acceleration and strain-based estimated displacement; second step is to find optimal value of neutral-axis position by defined minimization problem; the last step is applying RANSAC to remove outliers in the neutral-axis estimation.

3.1 Minimization Problem

Consider filtered response of strain-based estimated displacement u_i and acceleration acc_i at position x_i.

$$u_{f,i} = h(t) \otimes u_i(t) \tag{12}$$

$$acc_{f,i} = h(t) \otimes acc_i(t) \tag{13}$$

where *h(t)* is impulse response of band-pass filter to maintain only the first natural frequency. Based on the relationship between strain-based displacement and acceleration in Eq.(12) and Eq.(13), the neutral axis α can be obtained in time domain by minimizing the difference between second derivatives of the $u_{f,i}$ and $acc_{f,i}$ in a time interval, $T_1 \le t \le T_2$:

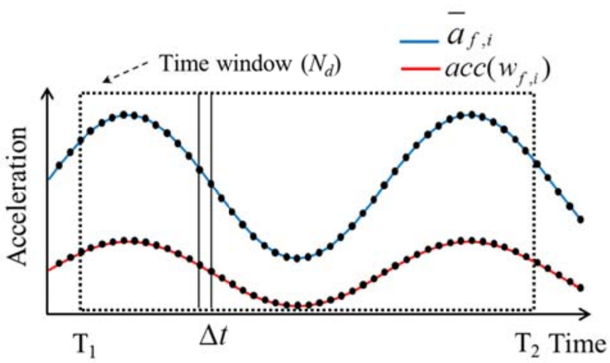

Fig. 1. Acceleration in a time interval T1 and T2

$$\underset{\mathbf{u}}{\text{Min}}\,\Pi = \frac{1}{2}\int_{T_1}^{T_2} \left(\frac{d^2 w_{f,i}}{dt^2} - \bar{a}_{f,i}\right)^2 dt \tag{14}$$

As strain and acceleration are discretized by a uniform time increment Δt , the objective function in Eq.(15) is expressed using the trapezoidal rule with an odd number of time steps.

$$\Pi \approx \frac{1}{2}(\frac{1}{2}(\frac{d^2w_1}{dt^2}-\bar{a}_1)^2+(\frac{d^2w_2}{dt^2}-\bar{a}_2)^2+\ldots+(\frac{d^2w_{N_d-1}}{dt^2}-\bar{a}_{N_d-1})^2$$
$$+\frac{1}{2}(\frac{d^2w_{N_d}}{dt^2}-\bar{a}_{N_d})^2)\Delta t=\frac{1}{2}(\frac{d^2w}{dt^2}-\bar{a})^T(L_a)^T La(a-\bar{a})\Delta t$$
$$=\frac{1}{2}\left\| L_a(\frac{d^2w}{dt^2}-\bar{a})\right\|_2^2 \Delta t \tag{15}$$

where N_d, a, $\bar{a}$ and $\|\square\|$ are the number of time window that contains discrete data in a time T_1 and T_2, second derivatives of the filtered estimated displacement and measured filtered acceleration. L_a is an identity diagonal matrix except the first and last entry of 1 $1/\sqrt{2}$. The N_d is three times the number of data corresponds to the first natural period (lee et al).

The second derivative of displacement is discretized by the central finite difference as in Eq.(16).

$$\frac{w_{f,i}(p+1)-2w_{f,i}(p)+w_{f,i}(p-1)}{(\Delta t)^2}=acc(w_{f,i}) \tag{16}$$

where $w_f(p)$ is the displacement at the *p*-th time step and *p* is discrete number from 1 to 2k+1. The Eq.(16) can be rewritten in a matrix form as Eq.(17).

$$\frac{1}{(\Delta t)^2}L_c w_{f,i}=\frac{d^2w_{f,i}}{dt^2} \tag{17}$$

The optimization function can be obtained as Eq.(18) by substituting Equation (17) into Eq.(15).

$$\underset{\mathbf{u}}{\text{Min}}\,\Pi=\frac{1}{2}\left\| L_a(L_c w_{f,i}-\Delta t^2\bar{a}_{f,i})\right\|_2^2 \tag{18}$$

The analytical solution for Eq.(15) is given in Eq.(19)

$$\alpha_i=(u_i^T L^T L u_i)^{-1}(u_i^T L^T L_a \bar{a}_i \Delta t^2) \tag{19}$$

The neutral-axis is susceptible to measuring *i*-th location of neutral axis, such that, for structures with uniform cross-section can have 1x1 matrix of α. The time window is marching by the size of N_d until it reaches the end of acceleration data, such that, α_i is extracted at every time window; it is then averaged for more accurate estimation.

3.2 RANSAC for Noise Reduction

Neutral-axis estimation as minimization problem can have many data for optimal estimation but can be noisy at which signal is relatively small. RANSAC removes outlier and provide the best model for the estimation. As neutral axis is constant value, parameteric model of y=α was used.

4 NUMERICAL SIMULATION ON A STEEL BEAM

Consider the simply supported beam of 50m modeled with 21 Euler-Bernoulli beam elements. The width and height of the beam are 2m and 5m. A moving load with a velocity of v = 1.5 m/s is applied vertically from the left to the right to generate non-zero mean displacement responses. The moving load consists of static load of 5000 kN and zero-mean Gaussian random load with a standard deviation of 500 kN. Time history analysis is conducted using MATLAB Simulink to simulate acceleration and strain for displacement estimation. The strain and acceleration responses were measured at N3, N8,N13 an N18.

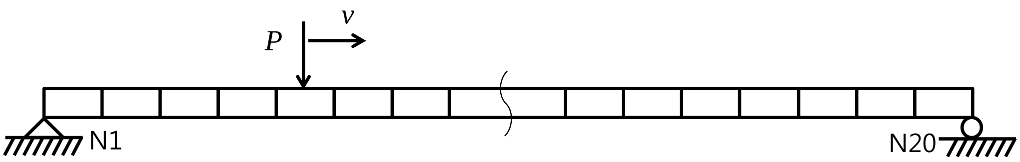

Fig. 2. Simply supported simple beam

4.1 Damage Location and Normalized Neutral Axis

Structural Damage was simulated from N1 to N20 in sequent, and neutral-axis at each stage was estimated and plotted in Fig 3. The magnitude of the damage was 10%, 30% and 50% loss of neutral axis at specific location on a beam. Fig 3(a) is the case where strain and acceleration are noise-free, while 10% noise and 5% noise in the strain and acceleration respectively was added for neutral-axis estimation plotted in Fig3b. In comparing Fig3(a) and (b), following summaries can be made:

1. The neutral-axis position changes as damage occurs and estimated result is more accurate at which damage occurred close to mid-span.
2. Measurement noise in strain and acceleration degrade estimation result, but the relationship between magnitude of the damage and corresponding change in neutral-axis was clear identified.
3. As the damage is very small, the actual loss of neutral-axis at local element was not accurately identified.

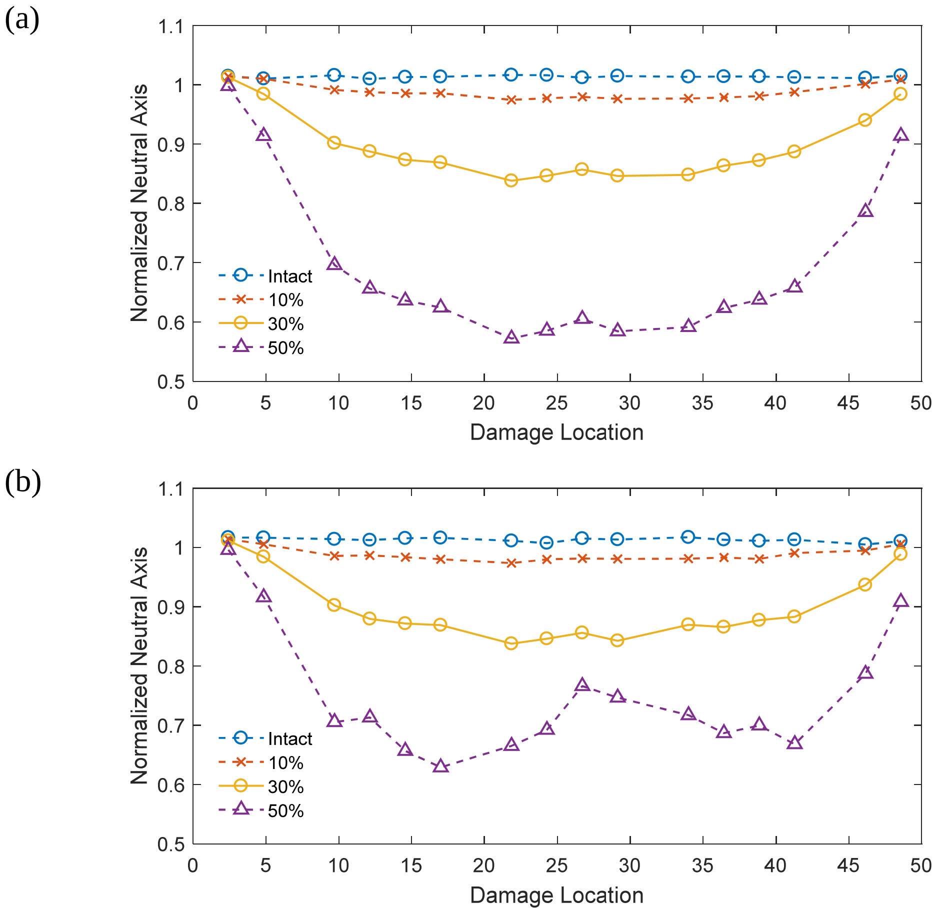

Fig. 3 Change in Neutral Axis due to Artificial Damage

5 CONCLUSIONS

The neutral axis can be used as damage indicator for structural damage such as reduction in stiffness. This paper proposes a methodology to estimate the neutral axis of a beam structure using acceleration and strain measurement. This paper first The proposed estimation method is composed of three stages. The acceleration and strain-derived displacement is filtered with band-pass filter designed to pass only the first natural frequency. Based on the filtered measurement, minimization between second-derivative of acceleration and strain-based displacement was formularized as minimization problem to obtain neutral-axis position α. The obtained neutral-axis is processed with RANSAC to eliminate any possible outliers in the estimation.

The proposed method is validated through numerical simulation using a simply supported beam. A reduction in height of every element of the beam was simulated to get the relationship between damage location and change in neutral axis. The simulation shows that the proposed neutral axis estimation can be used as damage indicator.

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