The 2017 World Congress on
Advances in Structural Engineering and Mechanics (ASEM17)
28 August - 1 September, 2017, Ilsan(Seoul), Korea

Assessment of the Separation Distance of Adjacent Buildings to Avoid Seismic Pounding

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ABSTRACT

Seismic pounding between adjacent buildings, especially those with different dynamic characteristics, is an inevitable phenomenon in earthquake-prone areas. This phenomenon, in buildings with inadequate separation distance, may result in local destruction or even overall collapse of structures. During the past, different combination rules have been adopted to determine the minimum distance between adjacent buildings, e.g. “sum of the absolute value of the displacement (ABS)”, “square root of sum of the squares (SRSS)” or “double difference combination (DDC)”. Both ABS and SRSS methods are based on the individual response of each system, and do not consider the correlation between the structural responses of adjacent systems. On the other side, the methodology of DDC rule is based on the linear modeling of two adjacent systems under the white noise and stationary input disturbance. In addition, ratio of the maximum displacement of two systems and, also, relative displacement to the corresponding standard deviation are assumed to be constant in DDC method. To overcome the weakness of the available methods, a rational relationship is presented in this paper. Both stationary and non-stationary processes are considered in the proposed method. Also, the main assumption of DDC method in considering a constant ratio of the average maximum displacement to the standard deviation is discussed and a correction is proposed. The method has been founded on analytical formulation in the frequency domain and enriched with the fundamentals of the random vibration theories.

Keywords: Random vibration theory, Seismic pounding, Separation distance, Stochastic analysis.

1. INTRODUCTION

Buildings, which may have different structural periods, may interact with each other during an earthquake if the adequate distance between the two structures is not provided. The importance of the seismic pounding can be studied by reviewing the amount of damage caused by this phenomenon during past earthquakes (Cole 2010).

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Given the importance of this issue, seismic design regulations offer the minimum distance between adjacent buildings necessary to avoid a collision. International codes and regulations apply the numerical methods of "sum of the absolute value of the displacement (ABS)", "the square root of sum of the squares (SRSS)" or "the double difference combination (DDC)" for determination of the required separation distance between structures. In the ABS method, the absolute response of the structures is used to determine the separation distance. Given the absolute values for the structural response of adjacent buildings, the symptom response is not considered, and therefore results of this method, especially when the periods of the two structures are close together, are conservative (Lopez Garcia 2004). The same studies have shown that the SRSS method will also yield conservative results in the cases where periods of two adjacent structures are close. However, results of this method are more reliable in comparison with those of ABS method. In other words, since both the ABS and SRSS methods evaluate the response of each of the two adjacent systems separately, and do not consider interactions of the two adjacent systems, they can be regarded as imperfect methods.

More reasonable methods for calculating the minimum allowable distance between buildings is the use of dual differential combination equation (DDC) (Jeng 1992). The DDC method, unlike the ABS and SRSS methods, considers the correlation coefficient between the two systems. Previous studies (Lopez Garcia 2004, 2009a) have shown that the DDC method provides reasonably accurate results regardless of whether the periods of two systems are close to each other or not. Even so, it has its own shortcomings, including the equation of the DDC method for two adjacent systems being considered linear, and the input excitations to the application of this method must be stationary processes with white noise density. In addition, the ratio of maximum displacement of both the system and the relative displacement to the corresponding standard deviation is assumed to be constant (Jeng 1992). For these reasons, the results of the DDC method have some error. The evaluation performed by Hong *et al*. (2003) showed that the results of the DDC method when the seismic excitation is modeled as a stationary random process, are somewhat conservative when the periods of the structures are close to each other, and slightly unconservative otherwise. The same results were obtained by Wang and Hong (2006) when the seismic excitation is modeled as a non-stationary random process. More recently, Garcia and Soong (2009b) found that the accuracy of the DDC rule depends not only on the ratio between the natural periods of two adjacent structural systems but also on the relationship between them and the period associated with the main frequency of the excitation.

Regarding the weakness of ABS, SRSS and DDC methods, this paper is aimed to develop a logical relationship for determining the separation distance between two adjacent systems. In the proposed method, two adjacent systems are considered non-linear, and the excitation density can be Kanai-Tajimi. Also, the assumption of a constant ratio of the average maximum displacement to the standard deviation is corrected; the proposed method is founded on a numerical solution and formulation in the frequency domain. The method is also enriched with the fundamentals of the random vibration theories and its accuracy is demonstrated by nonlinear dynamic analyses.

2. BASIC ASSUMPTIONS

This paper is aimed to develop a logical relationship for determining the separation distance between two adjacent systems with different forms of the structural behavior. To reach this aim, Bouc–Wen model has been selected in the present study. It is able to capture, in an analytical form, a range of shapes of hysteretic cycles (Weng 1980). The model's differential equation is as follows:

$$m\ddot{x}(t) + c\dot{x}(t) + \alpha kx(t) + (1-\alpha)kz(t) = P(t) \tag{1}$$

$$\dot{z} = A\dot{x} - \beta|\dot{x}||z|^{n-1}z - \gamma\,\dot{x}|z|^{n} \tag{2}$$

Eq. (1) is a differential equation of motion and Eq. (2) is a correlation between *x* and *z* displacements and virtual displacement. These simultaneously equations including: stiffness (*k*), mass (*m*), damping ratio (*c*), ratio of the stiffness after yield to the stiffness before yield (*α*) and parameters such as *n*, *β*, *γ* and *A* will tend to produce different models of hysteresis (Fig. 1).

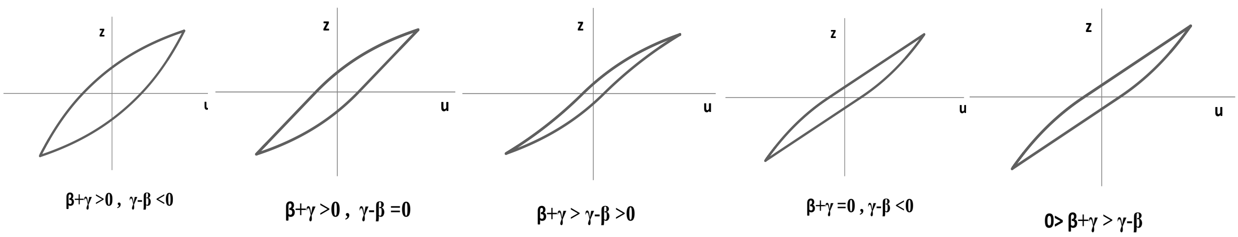

Fig. 1 Classical hysteresis models for different values of γ, β

Two adjacent systems "1" and "2" are modeled as classic Bouc-Wen SDOF systems (Fig. 2). The displacement response processes of the Bouc-Wen SDOF systems "1" and "2" are denoted by $x_1(t)$ and $x_2(t)$, respectively, and the relative displacement response process $x_{rel}(t)$ is given by:

$$x_{rel}(t) = x_1(t) - x_2(t) \tag{3}$$

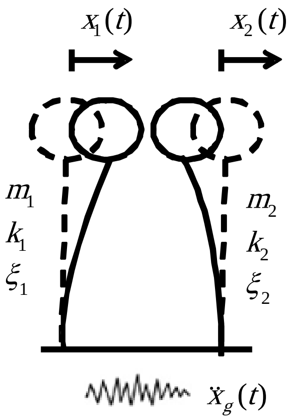

Fig. 2 Two SDOF systems

The minimum separation distance "*S*" required to avoid seismic pounding can be expressed by:

$$S = \max_t |x_{rel}(t)| \tag{4}$$

This extreme value highly depends on characteristics of earthquake records. So, several earthquake seismic records must be used for nonlinear time history analysis to consider record to record uncertainties. This numerical approach is time consuming, therefore the main aim of this research which is presented a closed form expression and efficient algorithm.

3. DESCRIPTION OF PROPOSED APPROACH

In this paper, the expected extreme value of relative displacement is considered as a separation distance. If the displacement response of adjacent systems, $x_1(t)$ and $x_2(t)$ are stationary Gaussian random processes with zero-mean, the mean square of relative displacement response process $x_{rel}(t)$ is equal to:

$$\begin{aligned} \sigma^2_{xrel} &= E[x^2_{rel}(t)] = E\left[(x_1(t) - x_2(t))^2\right]^2 \\ &= E[x_1^2(t)] + E[x_2^2(t)] - 2E[x_1(t)\,x_2(t)] \rightarrow \\ \sigma^2_{xrel} &= \sigma^2_{x1} + \sigma^2_{x2} - 2E[x_1(t)\,x_2(t)] \end{aligned} \tag{5}$$

where $E\{\}$ is the expected value, $\sigma^2_{x_1}$ and are the standard deviation of displacement of systems "1" and "2". The standard deviation of the displacement of the system "j" is equal to:

$$\sigma_{x_j}{}^2 = E[x_j{}^2] = \int_{-\infty}^{+\infty} S_{xx}(\omega)\, d\omega = \int_{-\infty}^{+\infty} |H_j(\omega)|^2 \; S_g(\omega)\, d\omega \tag{6}$$

and the covariance of (x_1, x_2) is defined as:

$$E\{x_1(t)\, x_2(t)\} = \int_{-\infty}^{+\infty} H_1(\omega)\, S_g(\omega)\, H_2(\omega)^* \, d\omega \tag{7}$$

In this equation, $H(\omega)$ is the frequency response of the BW model and $S_g(\omega)$ is input excitation density function. In this paper the seismic excitation is generated as a Gaussian, zero mean stationary random process $\ddot{u}_g(t)$ from Kanai-Tajimi power spectral density function, which is given by:

$$S_g(\omega) = \frac{\omega_g^4 + 4\xi_g^2\, \omega_g^2\, \omega^2}{(\omega_g^2 - \omega^2)^2 + 4\xi_g^2\, \omega_g^2\, \omega^2} S_0 \tag{8}$$

This model is a filtered white power spectral density (S_0) that has been passed over a layer of soil with a natural frequency of (ω_g) and damping ratio of (ξ_g).

Determination of $H(\omega)$ must be performed by partial derivative Eq. (1) and Eq. (2). These equations are nonlinear, and do not have an accurate solution. To achieve a closed form solution, a linearization approach must be applied. Parameters of the linear Bouc-Wen model are derived by the least square error theory (Wen 1980). The linearized form of Eq. (2) is:

$$\dot{z} = c_e\, \dot{x} + k_e\, z \tag{9}$$

The coefficients of linearized Bouc-Wen model are:

$$\begin{aligned} c_e &= A - \beta F_1 - \gamma F_2 \\ k_e &= -\beta F_3 - \gamma F_4 \end{aligned} \tag{10}$$

where,

$$\begin{aligned} F_1 &= \frac{\sigma_z^n}{\pi} \Gamma\left(\frac{n+2}{2}\right) 2^{n/2} I_s \\ F_2 &= \frac{\sigma_z^n}{\sqrt{\pi}} \Gamma\left(\frac{n+1}{2}\right) 2^{n/2} \\ F_3 &= \frac{n\,\sigma_{\dot{x}}\, \sigma_z^{n-1}}{\pi} \Gamma\left(\frac{n+2}{2}\right) 2^{n/2} \times \left\{ \frac{2\,(1-\rho_{\dot{x}z}^2)^{(n+1)/2}}{n} + \rho_{\dot{x}z} I_s \right\} \\ F_4 &= \frac{n}{\sqrt{\pi}} \rho_{\dot{x}z} \sigma_{\dot{x}}\, \sigma_z^{n-1} \Gamma\left(\frac{n+1}{2}\right) 2^{n/2} \end{aligned} \tag{11}$$

$$I_s = 2\int_{l}^{\pi/2} \sin^n \theta \, d\theta \, , \; l = \tan^{-1}\left(\frac{\sqrt{1-\rho_{\dot{x}z}^2}}{\rho_{\dot{x}z}^2}\right)$$

$$\sigma_z = \sqrt{E[z^2]}, \; \sigma_{\dot{x}} = \sqrt{E[\dot{x}^2]}, \; \rho_{\dot{x}z} = \frac{E[\dot{x}z]}{\sqrt{E[\dot{x}^2]\, E[z^2]}}$$

The amount of $H(\omega)$ is determined by using the coefficients of linearized Bouc-Wen model as follows:

$$|H_j(\omega)|^2 = \left| \frac{1}{(-\omega^2 + \alpha_j \omega_j^2 + \frac{(1-\alpha_j)\omega_j^2 \omega^2 c_{ej}}{\omega^2 + k_{ej}^2}) + i\,(2\xi_j \omega_j \omega - \frac{(1-\alpha_j)\omega_j^2 \omega c_{ej} k_{ej}}{\omega^2 + k_{ej}^2})} \right| \quad , \quad j = 1,2 \tag{12}$$

In Eq. (12), ω_j and ξ_j are the natural frequency and damping coefficient of Bouc-Wen model. With using the frequency response function, the variance of relative displacement is computable. For a zero-mean stationary Gaussian process, $x_{rel}(t)$, Davenport has shown the expected of the extreme values is given by the approximate relation (Davenport 1964):

$$S = \overline{X}_{rel} = ((2\ln(\nu T))^{0.5} + \frac{\gamma}{(2\ln(\nu T))^{0.5}})\,\sigma_{xrel} \tag{13}$$

$$\nu = \frac{\sigma_{\dot{x}rel}}{\pi\,\sigma_{xrel}} \tag{14}$$

Where *T* is a time duration, is Euler's constant equal to 0.5772. In the above equations, $\sigma_{\dot{x}rel}, \sigma_{xrel}$ are respectively, the standard deviations of the relative velocity displacement of two adjacent SDOF systems. Similarly, the mean square of relative velocity and the standard deviation of velocity of the system "*j*" are equal to:

$$\sigma_{\dot{x}rel}^2 = \sigma_{\dot{x}1}^2 + \sigma_{\dot{x}2}^2 - 2E[\dot{x}_1(t)\,\dot{x}_2(t)] \tag{15}$$

$$\sigma_{\dot{x}_j}{}^2 = E[\dot{x}_j{}^2] = \int_{-\infty}^{+\infty} \omega^2 S_{xx}(\omega)\, d\omega = \int_{-\infty}^{+\infty} \omega^2 |H_j(\omega)|^2 \; S_g(\omega)\, d\omega \tag{16}$$

and the covariance of $(\dot{x}_1, \dot{x}_2)$ is defined as:

$$E\{\dot{x}_1(t)\,\dot{x}_2(t)\} = \int_{-\infty}^{+\infty} \omega^2 H_1(\omega)\, S_g(\omega)\, H_2(\omega)^*\, d\omega \tag{17}$$

By putting Eq. (12) in Eq. (6) and Eq. (16), the displacement and velocity variance subject to white noise density function ($S_g(\omega) = S_0$), can be calculated using the residual method. Multiplying the numerator and the denominator of the frequency response function of Eq. (6) and Eq. (16) in $\omega^2 + k_{ej}^2$, gives:

$$\sigma_{xj}^2 = S_0 \int_{-\infty}^{+\infty} \left| \frac{-i\omega \; + k_{ej}}{\begin{array}{l}[-\omega^4 + i\omega^3(2\xi_j\omega_j) + \omega^2(-k_{ej}^2 + \alpha_j\omega_j^2 + (1-\alpha_j)c_{ej}\omega_j^2) \\ + i\omega(2\xi_j\omega_j k_{ej}^2 - (1-\alpha_j)\omega_j^2 c_{ej} k_{ej}) + \alpha_j\omega_j^2 k_{ej}^2]\end{array}} \right|^2 d\omega \quad , \; j=1,2$$
$$\sigma_{\dot{x}j}^2 = S_0 \int_{-\infty}^{+\infty} \left| \frac{i\omega(-i\omega \; + k_{ej})}{\begin{array}{l}[-\omega^4 + i\omega^3(2\xi_j\omega_j) + \omega^2(-k_{ej}^2 + \alpha_j\omega_j^2 + (1-\alpha_j)c_{ej}\omega_j^2) \\ + i\omega(2\xi_j\omega_j k_{ej}^2 - (1-\alpha_j)\omega_j^2 c_{ej} k_{ej}) + \alpha_j\omega_j^2 k_{ej}^2]\end{array}} \right|^2 d\omega \quad , \; j=1,2 \tag{18}$$

The value of integral of Eq. (18) according to the residual theorem is:

$$\sigma_{xj}^2 = \frac{\pi S_0\{A_0A_1 + 2k_{ej}^2 A_0A_3 + k_{ej}^4(A_1 + A_2A_3)\}}{-A_0(A_0A_3^2 - A_1^2 - A_1A_2A_3)} \quad , \; j=1,2$$
$$\sigma_{\dot{x}j}^2 = \frac{\pi S_0\{A_0(A_0A_3 - A_1A_2) + 2A_0A_1k_{ej}^2 + A_0A_3k_{ej}^4\}}{-A_0(A_0A_3^2 - A_1^2 - A_1A_2A_3)} \quad , \; j=1,2 \tag{19}$$

where,

$$\begin{cases} A_0 = \alpha_j^2\omega_j^2k_{ej}^2 \\ A_1 = 2\xi_j\omega_j k_{ej}^2 - (1-\alpha_j)\omega_j^2 c_{ej}k_{ej} \\ A_2 = k_{ej}^2 - \alpha_j\omega_j^2 - (1-\alpha_j)c_{ej}\omega_j^2 \\ A_3 = -2\xi_j\omega_j \end{cases} \quad , \; j=1,2 \tag{20}$$

With putting the displacement and velocity variance and covariance in Eq. (5) and Eq. (15), the separation distance is calculated by Eq. (13).

As shown above, the displacement and velocity variance of each system is calculated by Eq. (19), but the covariance of displacement and velocity (Eq. (7) and Eq. (17)) do not have an analytical form of solution, and the integral calculation including multiplication of several mixed fractions is not simple. Therefore, it is suggested that an approximate method in frequency domain be used to calculate the variance and covariance of the response. To achieve this solution, an equivalent linear SDOF system from Bouc-Wen model will be determined. Because of determination of the frequency and damping parameters of the equivalent linear system, there is no need to calculate an integral to determine the variance and covariance; also, the separation distance between the two nonlinear systems is simply calculated with Bouc-Wen behavioral model.

The linear SDOF system is considered equal to Bouc-Wen system with a frequency of ω_{eff_j} and damping ratio of ξ_{eff_j}. The standard deviation of the displacement and velocity of the linear system is equal to:

$$\sigma^2_{x\,eff_j} = E[x^2_{eff_j}] = \int_{-\infty}^{+\infty} \left| H_{eff_j}(\omega) \right|^2 S_0 \, d\omega =$$
$$= \int_{-\infty}^{+\infty} \left| \frac{1}{(-\omega^2 + \omega^2_{eff_j}) + i(2\xi_{eff_j}\omega_{eff_j}\omega)} \right|^2 S_0 \, d\omega = \frac{\pi S_0}{2\xi_{eff}\omega^3_{eff_j}} \quad , \; j = 1,2 \tag{21}$$

$$\sigma^2_{\dot{x}\,eff_j} = E[\dot{x}^2_{eff_j}] = \int_{-\infty}^{+\infty} \left| i\omega H_{eff_j}(\omega) \right|^2 S_0 \, d\omega =$$
$$= \int_{-\infty}^{+\infty} \left| \frac{i\omega}{(-\omega^2 + \omega^2_{eff_j}) + i(2\xi_{eff_j}\omega_{eff_j}\omega)} \right|^2 S_0 \, d\omega = \frac{\pi S_0}{2\xi_{eff}\omega_{eff_j}} \quad , \; j = 1,2 \tag{22}$$

The separation distance from the approximate frequency method is equal to the separation distance from the exact frequency method when the ratio of the standard deviation of the velocity to displacement for the system with the Bouc-Wen model is considered equal to ratio of the standard deviation of the velocity to displacement for an equivalent linear system. In this case, the Eq. (23) is obtained:

$$\frac{E[\dot{x}^2_{eff_j}]}{E[x^2_{eff_j}]} = \frac{E[\dot{x}^2_j]}{E[x^2_j]} \quad , \; j = 1,2 \tag{23}$$

If the standard deviation of the velocity and displacement for a system with Bouc-Wen model and an equivalent linear system in Eq. (23) are replaced, we will have:

$$\frac{\dfrac{\pi S_0}{2\xi_{eff_j}\omega_{eff_j}}}{\dfrac{\pi S_0}{2\xi_{eff_j}\omega^3_{eff_j}}} = \frac{\dfrac{\pi S_0\{A_0(A_0A_3 - A_1A_2) + 2A_0A_1k^2_{ej} + A_0A_3k^4_{ej}\}}{-A_0(A_0A_3^2 - A_1^2 - A_1A_2A_3)}}{\dfrac{\pi S_0\{A_0A_1 + 2k^2_{ej}A_0A_3 + k^4_{ej}(A_1 + A_2A_3)\}}{-A_0(A_0A_3^2 - A_1^2 - A_1A_2A_3)}} \rightarrow$$
$$\omega^2_{eff_j} = \frac{A_0(A_0A_3 - A_1A_2) + 2A_0A_1k^2_{ej} + A_0A_3k^4_{ej}}{A_0A_1 + 2k^2_{ej}A_0A_3 + k^4_{ej}(A_1 + A_2A_3)} \tag{24}$$

If the values of “*A*” (Eq. (20)) are replaced, the frequency of the linear system is given by:

$$\omega^2_{eff\,j} = \alpha_j\omega^2_j\left(1 + \frac{(1-\alpha_j)c_{ej}}{r_j^{\,2} + \alpha_j - 2\xi_{eff_j}r_j}\right) \quad , \; r_j = \frac{k_{ej}}{\omega_j} \tag{25}$$

By replacing the corresponding frequency in Eq. (22), and putting it equal to Eq. (19), the equivalent damping coefficient is equal to:

$$\xi_{eff_j} = \frac{\omega_j}{2\omega_{eff_j}} \left(\frac{c_{ej}(1-\alpha_j)(2\xi_j - r_j) + 2\xi_j(\alpha_j + r_j - 2\xi_j r_j)}{r_j^{\ 2} + \alpha_j - 2\xi_j r_j + (1-\alpha_j)c_{ej}} \right) \tag{26}$$

By determination of the parameters of the equivalent linear system of Bouc-Wen model, the correlation coefficient of the displacement and velocity for the two adjacent linear systems is obtained and therefore can be used. So, the standard deviation of each displacement, and relative displacement standard deviation in the approximate method is:

$$\sigma_{x1}^2 = \frac{\pi S_0}{2\xi_{eff_1}\omega_{eff_1}^3},\ \sigma_{x2}^2 = \frac{\pi S_0}{2\xi_{eff_2}\omega_{eff_2}^3}$$
$$E[x_1 x_2] = \rho_{x_1 x_2}\sqrt{\sigma_{x1}^2 \sigma_{x2}^2} \tag{27}$$
$$\rho_{x_1x_2} = \frac{8\sqrt{\xi_{eff1}\xi_{eff2}}(\xi_{eff1_1} + \xi_{eff2}\frac{\omega_{eff2}}{\omega_{eff1}})(\frac{\omega_{eff2}}{\omega_{eff1}})^{1.5}}{(1-(\frac{\omega_{eff2}}{\omega_{eff1}})^2)^2 + 4\xi_{eff1}\xi_{eff2}(1+(\frac{\omega_{eff2}}{\omega_{eff1}})^2)(\frac{\omega_{eff2}}{\omega_{eff1}}) + 4(\xi_{eff1}^2 + \xi_{eff2}^2)(\frac{\omega_{eff2}}{\omega_{eff1}})^2}$$

Also, the velocity standard deviation for each system, and the standard deviation of the relative velocity are equal to:

$$\sigma_{\dot{x}1}^2 = \frac{\pi S_0}{2\xi_{eff_1}\omega_{eff_1}},\ \sigma_{\dot{x}2}^2 = \frac{\pi S_0}{2\xi_{eff_2}\omega_{eff_2}} \tag{28}$$
$$E[\dot{x}_1 \dot{x}_2] = \rho_{\dot{x}_1 \dot{x}_2}\sqrt{\sigma_{\dot{x}1}^2 \sigma_{\dot{x}2}^2}$$
$$\rho_{\dot{x}_1\dot{x}_2} = \frac{\xi_{eff1}\omega_{eff2} + \xi_{eff2}\omega_{eff1}}{\xi_{eff1}\omega_{eff1} + \xi_{eff2}\omega_{eff2}} \frac{8\sqrt{\xi_{eff1}\xi_{eff2}}(\xi_{eff1_1} + \xi_{eff2}\frac{\omega_{eff2}}{\omega_{eff1}})(\frac{\omega_{eff2}}{\omega_{eff1}})^{1.5}}{(1-(\frac{\omega_{eff2}}{\omega_{eff1}})^2)^2 + 4\xi_{eff1}\xi_{eff2}(1+(\frac{\omega_{eff2}}{\omega_{eff1}})^2)(\frac{\omega_{eff2}}{\omega_{eff1}}) + 4(\xi_{eff1}^2 + \xi_{eff2}^2)(\frac{\omega_{eff2}}{\omega_{eff1}})^2}$$

When the excitation has Kanai-Tajimi pattern, Eq. (27) and Eq. (28) will not change. Because of that, the value of frequency response function for the Bouc-Wen system in frequency of zero is maximum. Hence, by using the approximation, we can calculate the value of integral of multiplying the frequency response function in the Kanai-Tajimi model. The value is equal to the multiplication of the integral of the only frequency response function by the value of Kanai-Tajimi function at zero frequency.

4. RESULTS

Structures with different frequencies were taken into account to verify the approximate equations, and the results of the proposed Eq. (27) were compared with those of the precise integration (Eq. (19)). These results were given for the white wave pattern having S_0 = 100. The frequency of the first structure in these calculations is constant and equal to 7 rad/s, and the second system's frequency is equal to 8, 9 and

10. The remaining parameters of Bouc-Wen are considered to be constant (A = 1, n = 1, γ = -1, β = 2, α = 0.05). The variance, covariance, and correlation coefficient values in frequency domain for the two systems under excitation by white noise function, which are resulted from the two approximate and exact methods are listed in Table (1).

Table 1 Comparing the frequency domain (precise integration) with the proposed method in terms of the calculation of the relative displacement variance under white noise

Frequency		frequency domain				proposed method			
ω_1	ω_2	σ_{x1}	σ_{x2}	$E[x_1 x_2]$	σ_{xrel}	σ_{x1}	σ_{x2}	$E[x_1 x_2]$	σ_{xrel}
7	7	6.01	4.39	26.1	1.77	5.98	4.4	25.7	1.91
	8		3.04	17.4	3.26		3.08	18.5	3.18
	9		2.66	15	3.62		2.6	13	4.1

With respect to Table (1), a good agreement is observed between the results of the approximate method and those of the exact method obtained using exact integration. Concerning the applicability of the proposed method, as a replacement of the exact integration, Eq. (27) can be employed instead of Eq. (19) for an excitation input with white noise density. Additionally, to verify the approximate relationships for the Bouc-Wen system under excitation by the Kanai-Tajimi density, the three pairs of previous systems were considered, and then the results of this model (Eq. 27) were compared with those of exact integration. The resulted values for the Kanai-Tajimi model are as follows: S_0 = 100, ξ_g = 0.34, ω_g = 18.34. The variance, covariance, and correlation coefficient obtained using the proposed and precise integration methods for the pairs of adjacent systems under excitation by Kanai-Tajimi density function are listed in Table (2).

Table 2 Comparing the frequency domain (precise integration) with the proposed method in terms of the calculation of the relative displacement variance under the Kanai-Tajimi model

Frequency		frequency domain				proposed method			
ω_1	ω_2	σ_{x1}	σ_{x2}	$E[x_1 x_2]$	σ_{xrel}	σ_{x1}	σ_{x2}	$E[x_1 x_2]$	σ_{xrel}
7	8	7.36	4.76	30	1.78	6.29	4.69	29	1.89
	9		3.7	22.8	2.93		3.63	21.4	3.16
	10		2.99	18	3.69		2.9	15.8	4.05

According to Table (2), a good agreement exists between the results of the approximate method and those of the exact method obtained from exact integration. In terms of the applicability of the proposed method, the proposed equations can be used instead of the exact integration method.

A number of nonlinear dynamic analyses can be used with the purpose of verifying the calculations associated with the mean maximum relative displacement. In this method, two linear SDOF systems are subjected to 300 records produced by MATLAB with the Kanai-Tajimi density, and the response of each system at each moment is determined. The relative displacement defined as the difference between the displacements of two systems is determined at each moment, and the maximum relative displacement for each specific record is then determined. In this case, the mean maximum relative displacement is regarded as the mean of the maximum displacements obtained from 300 records.
The three pairs of systems with specifications given in Table (2) were considered in order to verify the equations proposed for calculating the mean maximum relative displacement, and the results of the exact integration and the proposed method, both existing in the frequency domain, were compared with the results from the time domain. The corresponding results are provided for the Kanai-Tajimi model with ξ_g = 0.34, ω_g = 18.34, S_0 = 67.5. Table (3) presents the maximum relative displacement for the two adjacent systems.

Table 3 Comparison of mean maximum displacement obtained from the solution in time and frequency domains

Frequency of system (1)	Frequency of system (2)	Expected extreme value of relative displacement		
		300 nonlinear dynamic analysis	Solution in frequency domain (precise integration)	proposed method in frequency domain
7	8	4.98	4.77	4.86
	9	8.3	8	8.23
	10	10.6	10.1	10.6

The proposed method which employs the approximate equation is capable of calculating the mean maximum relative displacement between two nonlinear systems, as can be seen in Table (3), and the results of 300 nonlinear dynamic analyses are obtained just by one equation. The proposed method lacks the flaws of DDC, since the excitation density can be Kanai-Tajimi; both systems are regarded as nonlinear behavioral models; and the assumption concerning a constant ratio of the mean maximum displacement to the standard deviation is modified.

5. CONCLUSIONS

Seismic design regulations state the minimum distance between adjacent buildings required to avoid collision (ABS, SRSS and DDC method), which is based on the absolute response of two adjacent structures. Unlike the ABS and SRSS methods, the DDC method takes the correlation coefficient between two systems into account; however, it has some drawbacks including the consideration that the DDC method's equation for two adjacent systems be linear. Furthermore, the input excitations must be processes with white noise density to be able to utilize this method. In addition, the ratio of the maximum displacement of both systems as well as the relative displacement to their corresponding standard deviation is assumed constant. In this research, to improve the DDC method, based on the random vibration theory, a method is proposed. In the proposed method, two adjacent systems are considered nonlinear Bouc-Wen models, the excitation density can be both Kanai-Tajimi and white noise density, and also the assumption of a constant ratio of the mean maximum displacement to the standard deviation is corrected. Moreover, the separation distance is determined based on the random vibration theory in the frequency domain, and its precision is then verified by various nonlinear dynamic analyses. It was shown that the proposed relations can be used to directly compute the mean and variance of separation distance for two nonlinear adjacent systems with a good approximation.

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