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An analytical solution to assess the stresses in vertical backfilled stopes considering an arc layer element

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ABSTRACT

Stress estimation is a critical task for the design of backfilled openings. Over the years, a number of works have been published on the stress analyses in backfilled stopes using Marston model proposed in the years 1930. In the 1980s, Handy suggested the use of parabola layer element while Harrop-Williams found that Handy's layer element can be approached by circular arc layer element. In this paper, an analytical solution is developed to assess the stresses in vertical backfilled openings by considering a circular arc layer element. The proposed solution is compared with numerical modelling.

1. INTRODUCTION

In the design of backfilled openings, stress estimation is a critical concern. This task is commonly performed by using the Marston (1930) solution, which was in turn obtained by considering the global equilibrium of a horizontal layer element. However, shear stresses are generally present in places away from the center of the opening and the vertical stress is generally not uniform across the width of the opening. Subsequently, the local equilibrium at a place away from the center was not considered and the ensuing vertical stress obtained by the Marston (1930) solution is only an average value, which does not represent the stress at the center and close to the walls of the opening. To overcome this limitation, Handy (1985) proposed to consider a layer element along which shear stress is absent and the principal stresses are uniformly distributed. Based on the numerical analyses of Sokolovoski (1965), Handy (1985) proposed to consider the equilibrium of a parabola layer element. Later, Harrop-Williams (1989) found that the Handy's parabola element can closely be approached by a circular arc layer element. Nevertheless, no analytical solution was proposed for

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estimating the stresses in backfilled opening by considering parabola or circular arc layer element. An analytical solution was proposed by Singh et al. (2011) by considering circular arc layer element for estimating the stresses in inclined backfilled stopes. Their formulation was expressed in a local coordinate system rather than in a global Cartesian system. Transformation is necessary to obtain the stresses at a point away from the inclined center line.

In this paper, an analytical solution is presented for estimating the stresses in a vertical backfilled stope by considering the equilibrium of a circular arc layer element (Harrop-Williams 1989; Singh et al. 2011). The proposed solution is compared with existing solutions and numerical modeling.

2. FORMULATION

Fig. 1 shows a vertical backfilled opening having a width of 2*B*. Bases on the distribution of the minor principal stresses in a backfilled opening, obtained numerically by Sokolovoski (1965), Handy (1985) proposed to describe the shear stress-free layer element as a parabola, which was later approached by Harrop-Williams (1989) as a circular arc element:

$$X^2 + \left(Y - \frac{1}{\lambda}\right)^2 = \left(\frac{1}{\lambda}\right)^2 \quad (1)$$

where *X* and *Y* are the abscise and ordinate of the local system associated with the arc, respectively (Fig. 1); λ is the curvature of the circle, further expressed as follows by Harrop-Williams (1989):

$$\lambda = \frac{1}{R} = \frac{1}{\kappa B} \quad 2)$$

where *R* is the radius of the circle; κ (= *R*/*B*) is a ratio of the circle diameter (2*R*) to the stope width (2*B*), further defined as follows by Harrop-Williams (1989):

$$\kappa = \frac{1}{\sin(45° - \phi/2)} \quad (3)$$

where ϕ is the friction angle of the backfill.

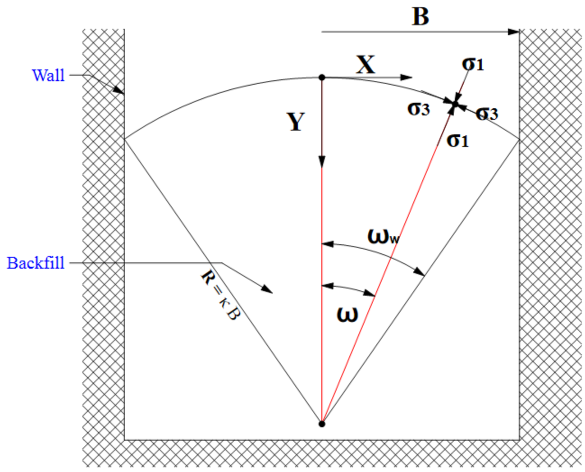

Fig. 1. An arc diagram of vertical backfilled opening (after Harrop-Williams 1989)

The consideration of the equilibrium of the arc layer element leads to the following expression for the major principal stress along the circular arc element:

$$\sigma_1 = \frac{Q}{S}\left(1 - exp\left(-\frac{S}{P}(z)\right)\right) \tag{4}$$

where *P*, *Q* and *S* are given as follows:

$$P = B\,\kappa\,\left[\sin\omega_w\right] \tag{5}$$

$$Q = \left(\gamma\,B\kappa\,\omega_w - \frac{c}{\cos\omega_w}\right) \tag{6}$$

$$S = \left[sin^2\,\omega_w + K_{ps}\,cos^2\,\omega_w\right]\frac{tan\,\delta}{\cos\omega_w} \tag{7}$$

where γ is the unit weight of the backfill; K_{ps} is a ratio of the minor to major principal stresses; ω_w is an angle between the major principal stress σ_1 and the vertical wall; *c* and δ are the cohesion and friction angle along the interfaces between the backfill and rock walls, respectively. In civil engineering with planar walls, the value of δ is usually close to two third of the backfill friction angle. For mine stopes with rough walls, its value is usually close to the friction angle of backfill because shearing mostly takes place within the backfill.

At a given point, the principal major stress expressed in the global Cartesian system (*x*, *y*) becomes:

$$\sigma_{1(x,y)} = \frac{Q}{S}\left[1 - exp\left(-\frac{S}{P}\left(y - \left[B\kappa - \sqrt{(B\kappa)^2 - x^2}\right]\right)\right)\right] \tag{8}$$

The vertical and horizontal stresses at a given point (*x*, *y*) are then expressed as follows:

$$\sigma_{v(x,y)} = \sigma_{1(x,y)} \left[1 - \left(1 - K_{ps}\right)\left(\frac{x}{B\kappa}\right)^2\right] \tag{9}$$

$$\sigma_{h(x,y)} = \sigma_{1(x,y)} \left[K_{ps} + \left(1 - K_{ps}\right)\left(\frac{x}{B\kappa}\right)^2\right] \tag{10}$$

3. PROPOSED SOLUTION

Some preliminary comparisons between the analytical solution (Eqs. 8 to 10) and numerical modeling show that Eq. (8) needs to be adjusted. The adjustment leads to the following equation:

$$\sigma_{1(x,y)} = \frac{Q'}{S}\left[1 - \exp\left(-\frac{S}{P'}\left(y - \left[B\kappa - \sqrt{(B\kappa)^2 - x^2}\right]\right)\right)\right] \tag{11}$$

where *P*' and *Q*' are expressed as follows:

$$P' = 2.14\ (\tan\phi)^{0.25}\ B\ \kappa\ \left[\sin \omega_w\right] \tag{12}$$

$$Q' = 2.14\ (\tan\phi)^{0.25}\ \gamma\ B\kappa\ \omega_w\ - \frac{c}{\cos \omega_w} \tag{13}$$

Eqs. (9) to (13) constitute the proposed analytical solution for estimating the horizontal and vertical stresses in vertical backfilled opening.

4. COMPARISON BETWEEN ANALYTICAL SOLUTION AND NUMERICAL MODELING

Fig. 2 shows a comparison of the stresses along the vertical center line of the opening between the proposed analytical solution (Eqs. 9 to 13) and the numerical results used to obtain the modified equations (i.e. the proposed solution). Without any surprise, the agreement between the proposed solution and numerical modeling is excellent.

To test the validity of the proposed analytical solution, additional numerical modeling has been done. Fig. 3 shows a comparison between the vertical (Fig. 3a) and horizontal (Fig. 3b) stresses along the vertical center line of the opening, obtained by numerical modeling and predicted with the proposed analytical solution. One sees that the agreement between the numerical and analytical solutions remain very good. The proposed solution can thus be used to estimate the stresses along the vertical centre line of vertical backfilled openings.

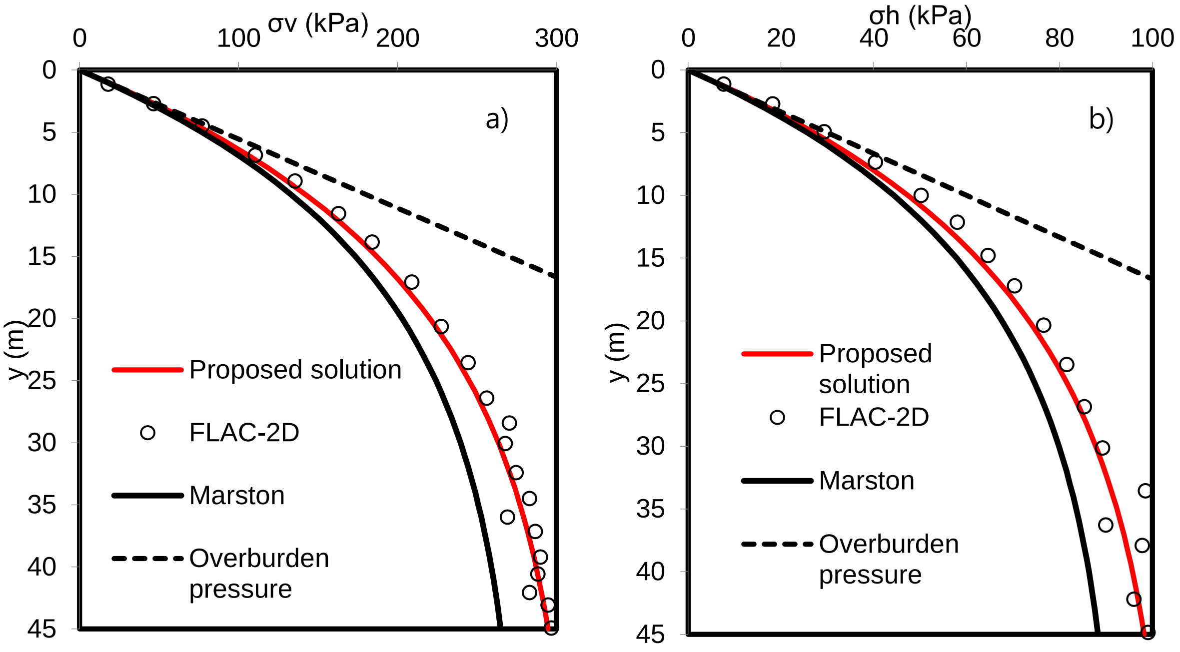

Fig. 2. Variation of the (a) horizontal and (b) vertical normal stresses along the VCL, obtained by the proposed solution (Eqs. 9 to 13) and numerical modeling for *H*=45 m, ø = 30° and B = 6 m.

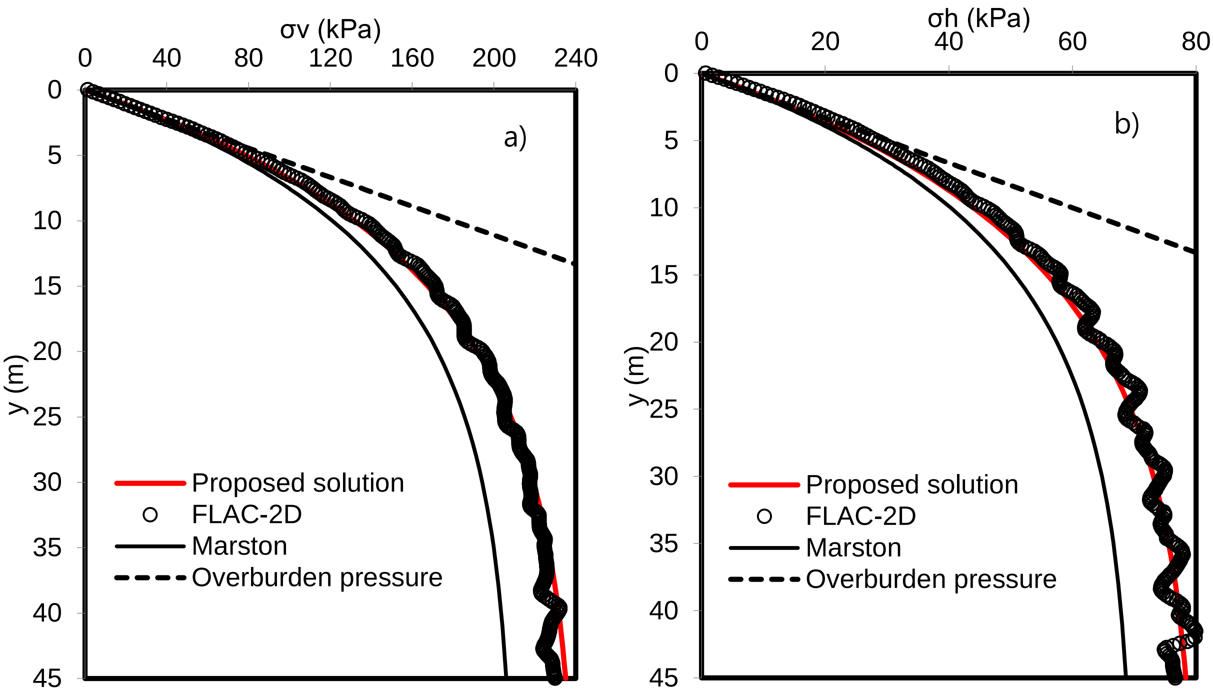

Fig. 3. Variation of the (a) horizontal and (b) vertical normal stresses along the VCL, obtained by the proposed solution (Eqs. 9 to 13) and numerical modeling for *H*=45 m, ø = 30° and *B* = 4.5 m.

5 CONCLUDING REMARKS

In this paper, an analytical solution was presented after considering a circular arc element proposed by Harrop-Williams to estimate the stresses in a vertical backfilled opening. The expressions were modified against numerical modeling. The validity of the final proposed analytical solution was tested by comparing the vertical and horizontal stresses along the vertical center line of the opening, obtained by additional numerical modeling and calculated by the analytical solutions. The results showed that the proposed solution can be used to estimate the stresses along the vertical center line of vertical backfilled openings (stopes, trenches, silos, etc.).

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