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A simplified model for a small propeller with different airfoils along the blade

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ABSTRACT

The flow through the rotor of a propeller is complex due to the rotor wake interaction. Varieties of methods with different simplifications were used to represent the flow field and its interaction with the rotor. The impact of these simplifications on the precision of the predicted results and computation time were evaluated. These methods include momentum theory, blade element theory, lifting line theory, finite volume methods, panel methods, boundary element methods and CFD analysis. The objective of the present work is to propose a robust and simple method to calculate the aerodynamic characteristics of the propeller with relatively good precision. Both the momentum theory and blade element theory were used in home built FORTRAN code and the predicted results were validated against results from the Panel method and available experimental measurements showing acceptable agreement. Furthermore, the numerical code was used to investigate the effect of incorporating three different blade sections on the power and thrust coefficients.

1. INTRODUCTION

Small and medium size rotors are used in many recent applications as in propulsion of small airplanes, unmanned aerial vehicles (UAV), autonomous underwater vehicle (AUV), and small wind turbines and ducted propellers. Understanding rotor action and interaction with wake flow field are important aspects to better formulate and predict the performance of propeller rotors and wind turbines. Momentum theory applied to rotors and blade element theory were widely used for light loaded blades. Theodorsen (1948) developed the propeller theory with ideal load distribution from the dynamics of the wake vortex sheet. This condition of maximum propeller efficiency occurs when the wake vortex sheets are helicoidal without any deformation.

The flow over the rotors is very complex due to the circular movement of the blades and the strong interaction with the wake (Palmiter and Katz 2010). For this reason, the precise calculation of the aerodynamic behavior of the rotor depends on the

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correct modeling of the rotor wake, whose complex structure limits pure analytical methods and hence the numerical methods are inevitably necessary (Dumitrescu and Cardos 1998).

Dumitrescu and Cardos (1998) reported that with the increase of interest in wind energy, intensive research on the aerodynamic behavior of wind turbines has been conducted during the past decades. Techniques for the prediction of aerodynamic loading and performance of wind turbines have been reviewed as in De Vries (1979; 1983). As a result numerous combinations of analytical and numerical models that can describe well and predict accurately the horizontal windmill performance were elaborated. One of the major difficulties is the ability to simulate well the rotor wake and still have a numerical solution with low computational cost.

There are two main approaches to model the rotor wake. The first is to assume a prescribed wake where the wake is known a priori and hence the velocity field. This allows calculating the corresponding induced velocity and circulation distribution along the blade.

The second method is based upon assuming a free-wake. In the method the initial geometry of the vortex is assumed, being composed of discrete vortex elements which are allowed to convect in the velocity field until the wake reaches equilibrium state. This method needs a tremendous computing time. In view of this many investigators looked for alternatives to reduce the computing time as in Miller (1983) and Afjeh and Keith (1986).

Dumitrescu and Cardos (1998) used a lifting line method to replace the wind turbine blades with the trailing vortices shed along the turbine blade. The model is nonlinear and was solved iteratively. The performance parameters were calculated by the Biot-Savart law and the Kutta-Joukowski theorem.

Palmiter and Katz (2010) used a tridimensional potential flow based panel code to model the flow over rotating propeller blades. They modified an existing panel code and studied the wind turbine and propeller flows and validated their predictions with available results.

Khan and Nahon (2014) presented a slipstream model which uses simple analytical and semi-empirical equations. The numerical predictions were found to agree with experimental data.

The objective of the present work is to develop a robust and simple method to calculate the aerodynamic characteristics of the propeller with relatively good precision. Both the momentum theory and blade element theory were used in home built FORTRAN code and the predicted results were validated against results from the panel method and available experimental measurements showing acceptable agreement. Furthermore, the numerical code was used to investigate the effect of using a blade composed of three different airfoil sections on the power and thrust coefficients.

2. FORMULATION

The propeller develops an axial force called thrust T at an advance V for a rotational n due to a torque Q. In this manner the propeller efficiency is the ratio of the useful power to the power input P

$$\eta = TV/(2\pi nQ), \tag{1}$$

The aerodynamic characteristics are usually expressed by no dimensional forms which depend on the Reynolds and Mach numbers based on the blade tip velocity and the advance ratio J. The advance ratio is defined as

$$J = V/(nD), \tag{2}$$

while the tip velocity is given by

$$V_{tip} = \pi nD, \tag{3}$$

The thrust k_T, torque k_Q and the power k_P coefficients are defined as

$$k_T = T/(\rho n^2 D^4), \tag{4}$$

$$k_Q = Q/(\rho n^2 D^5), \tag{5}$$

$$k_P = P/(\rho n^3 D^5) = 2\pi k_Q, \tag{6}$$

In terms of these coefficients the efficiency of the propeller can be alternatively written as

$$\eta = J\, k_T / k_P, \tag{7}$$

As mentioned before, the calculation routine to predict the general performance of the propeller associates the momentum theory due to Rankine and Froude (Wald 2006) with Glauert blade element theory (Glauert 1926). The Froude momentum theory treats the propeller as a disc with an axial velocity in relation to the disc given by the advance velocity V corrected by the inflow factor a,

$$V_0 = V(1+a), \tag{8}$$

From the linear momentum conservation and the pressure difference across the disc, the velocity component well behind the disc V_S is

$$V_S = V(1+2a), \tag{9}$$

Glauert blade element theory for propellers indicates that the velocity component in the plane of rotation V_w can be calculated from

$$V_w = 2\pi n(1-b)r\,, \tag{10}$$

where b is the swirl factor which accounts for the effects of the wake vortex system on the flow angular velocity in the rotor plane.

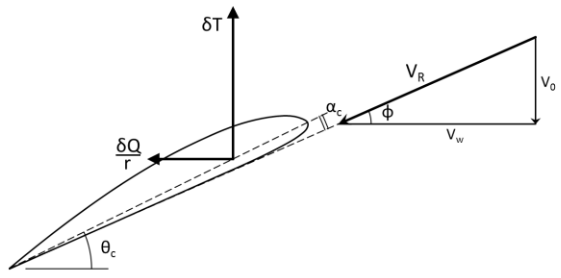

Fig. 1 Cross section of the rotor blade at radius *r* indicating the velocity components and the acting forces

Applying the principle of conservation of linear momentum to the flow in an infinitesimal radial ring one can determine the elementary thrust as

$$\frac{dT}{dr} = 4\pi\rho r V^2 a(1+a)\,, \tag{11}$$

Similarly, the elementary torque is given by

$$\frac{dQ}{dr} = 4\pi\rho r^3 V b(1+a)2\pi n\,, \tag{12}$$

The elementary thrust and torque can be obtained alternatively from the lift and drag acting on the blade element as below

$$\frac{dT}{dr} = Bc\frac{1}{2}\rho V_R^2\left(C_\ell\cos\phi - C_d\sin\phi\right), \tag{13}$$

$$\frac{dQ}{dr} = Bcr\frac{1}{2}\rho V_R^2\left(C_\ell\sin\phi + C_d\cos\phi\right), \tag{14}$$

where B is the number of blades, V_R is the resultant velocity with reference to the blade and ϕ is the angle of the vector V_R with the plane of rotation of the propeller.

The local resultant velocity V_R is given by

$$V_R = V_0/\sin\phi = V_w/\cos\phi\,, \tag{15}$$

while the angle ϕ is calculated from

$$\phi = \tan^{-1}\left[\frac{V_0}{V_w}\right] = \tan^{-1}\left[\frac{J}{2\pi(r/D)}\left(\frac{1+a}{1-b}\right)\right], \tag{16}$$

The angle of attack α_c is the angle between the resultant velocity V_R and the chord, calculated as the difference between the pitch angle (twist) of the blade section θ_c and the angle ϕ of the resultant velocity. The pitch angle θ_c is defined as the angle between the local chord and the plane of rotation of the propeller. The geometric pitch p of the blade section is related to the pitch angle θ_c by the expression $p = 2\pi\, r \tan\theta_c$.

The geometry of the blade is usually given by the distributions of chord, thickness and the geometrical pitch as function of radius. The pitch angle of the blade is defined as the pitch angle of the blade section localized at $r = 0.75R$, where R is the radius of the blade tip, $R = D/2$.

Equating Eq. (11) to Eq. (13) and Eq. (12) to Eq. (14) one can obtain

$$a = (1+a)\frac{\sigma}{4\sin^2\phi}\left(C_\ell\cos\phi - C_d\sin\phi\right), \tag{17}$$

$$b = (1+a)\frac{\sigma}{4\sin^2\phi}\left(C_\ell\sin\phi + C_d\cos\phi\right)\frac{J}{2\pi(r/D)}, \tag{18}$$

where σ is the solidity defined as $\sigma = (Bc)/(2\pi r)$.

Eq. (17) and Eq. (18) can be used to calculate the factors a and b iteratively.

The thrust loading coefficient δk_T of the blade element δr at r is given by

$$\delta k_T = \frac{\frac{dT}{dr}}{B\rho n^2 D^4}\,\delta r = \frac{1}{2}c\left[\frac{J(1+a)}{D\sin\phi}\right]^2\left(C_\ell\cos\phi - C_d\sin\phi\right)\delta r\,, \tag{19}$$

While the torque loading coefficient of the blade δk_Q is given by

$$\delta k_Q = \frac{\frac{dQ}{dr}}{B\rho n^2 D^5}\,\delta r = \frac{1}{2}c\left[\frac{J(1+a)}{D\sin\phi}\right]^2\left(C_\ell\sin\phi + C_d\cos\phi\right)\frac{r}{D}\,\delta r\,, \tag{20}$$

The coefficients of thrust k_T and the torque k_Q of the propeller can be obtained by integrating Eq. (19) and Eq. (20).

One can see that the radial loading coefficients of the blade δk_T and δk_Q depend only the propeller geometry and the advance ratio J. This also applies to the coefficients of thrust k_T, torque k_Q and power k_P of the propeller.

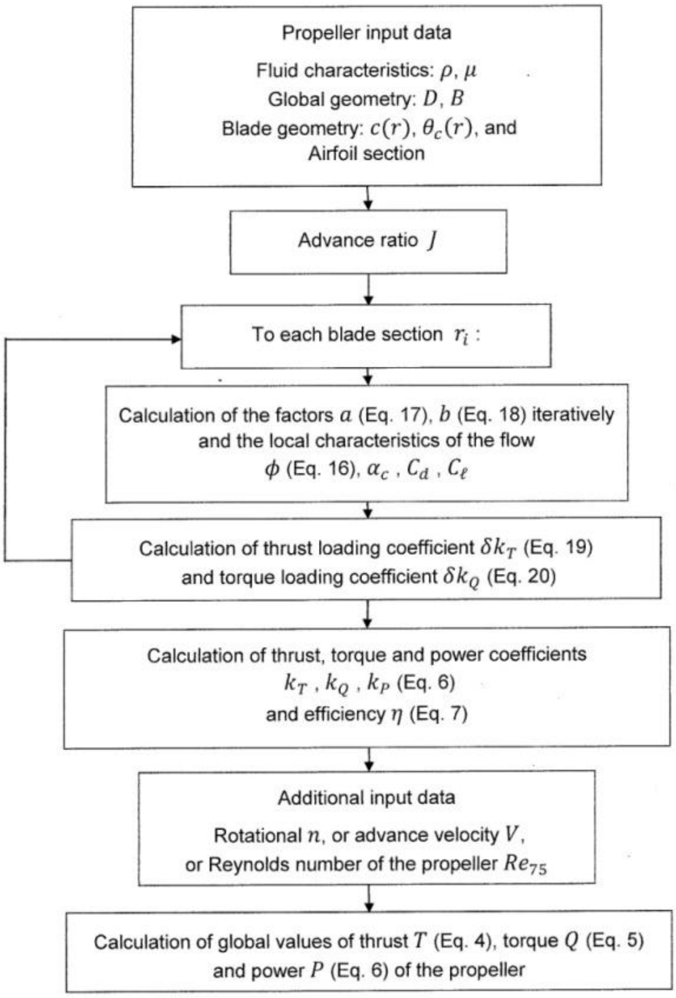

Fig. 2 Block diagram of the propeller calculation procedure

Hence the performance characteristics of the propeller can be presented in terms of k_T, k_Q, k_P and η in terms of the advance ratio J. The specification of the value of the advance velocity V or the rotation n determine the Reynolds number of the propeller Re_{75} and allows calculation the global values of thrust T, torque Q and power P.

The procedure to dimension a propeller and evaluate its performance parameters is presented in the block diagram presented in Fig. 2.

3. VALIDATION

The method adopted here is simple and can be used to pre dimension a light loaded propeller and determine its characteristics. To establish the validity of this method and its viability in calculating the propeller performance and/or its pre dimensioning, the numerical predictions from the present method are compared with experimental results (Hartman and Biermann 1938) and numerical predictions based on panel method (Palmiter and Katz 2010).

The available experimental results are for the propeller Clark Y 5868-9, with airfoil Clark Y, diameter 10 ft, two blades and for two blade pitch angles of 25° and 35° (Hartman and Biermann 1938). Palmiter and Katz (2010) realized recent numerical results for the same propeller, they used three dimensional panel method.

Fig. 3 shows a comparison of coefficient of thrust k_T predicted from the present method compared with the experimental results and with the numerical predictions calculated by the panel method. As can be seen the agreement is good for the case of pitch angle of 25°. When the pitch angle is increased to 35°, there is noticeable divergence between the present predictions and the experimental results for low advance ratios due to possible flow separation (Palmiter and Katz 2010), which is not accounted for in the both numerical methods. They justified the low values calculated by the panel method due to considering rigid wake to reduce the computational time.

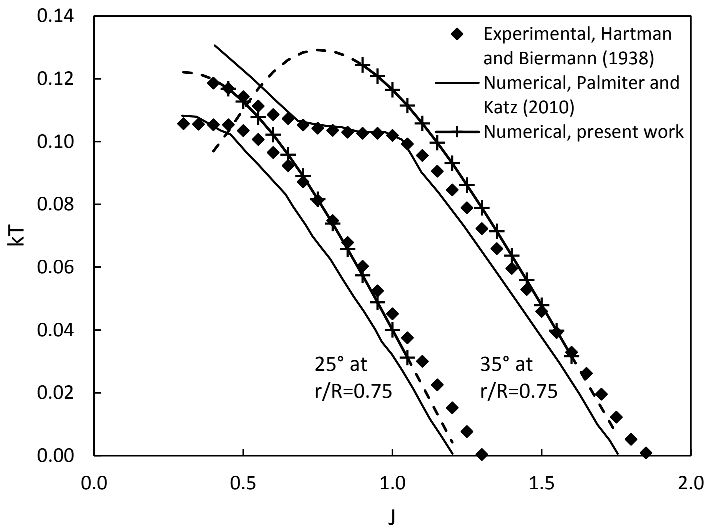

Fig. 3 Variation of the thrust coefficient with the advance ratio for propeller Clark Y 5868-9

Fig. 4 shows a comparison between the present predicted values of coefficient of power k_P, the experimental results and the numerical results due to Palmiter and Katz (2010). It is possible to verify that the results from the present method are closer to the experimental results. As can be seen the agreement between the present predictions and experiments are good except for values of J below 1.0 for the case of pitch angle of 35°. In this region, both numerical predictions indicate noticeable divergence in comparison with experiments.

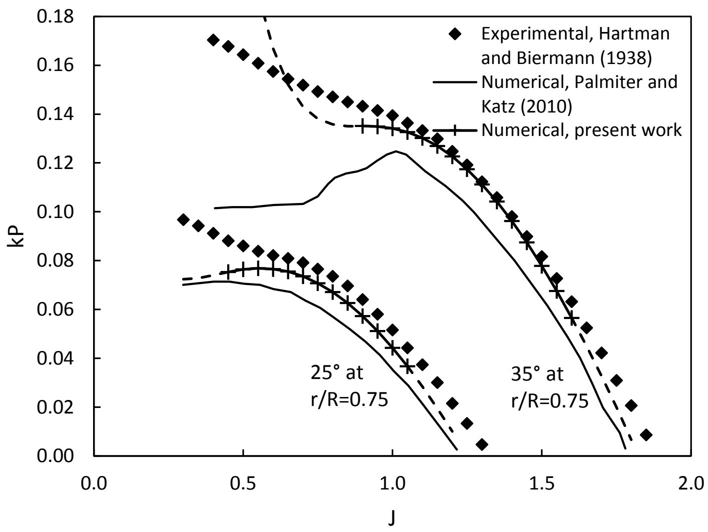

Fig. 4 Variation of the power coefficient with the advance ratio for propeller Clark Y 5868-9

Fig. 5 shows the efficiency of the propeller predicted from the present method compared with experimental results and numerical ones due to Palmiter and Katz (2010). As can be seen the agreement is good with the results of panel method. However, both numerical methods overestimate the efficiency.

It is interesting to observe that around the advance ratio for best efficiency, that is, around $J = 0.95$ ($\eta = 0.91$) for the case of pitch angle of 25^{o} and around $J = 1.4$ ($\eta = 0.93$) for the pitch angle of 35^{o}, the present predicted results are good for both coefficients k_T and k_P as can be verified from Fig. 3 and Fig. 4. These results are significant for a simple rapid method in comparison with other elaborate methods.

Table 1 presents the aerodynamic characteristics calculated by the present method for the advance ratio which produces maximum efficiency for two blade propellers Clark Y 5868-9 with pitch angle of 25° and 35°. The numerical predictions are compared with the experimental results from Hartman and Biermann (1938). One can observe that the relative error is around 7% for the thrust coefficient and of order of -12% and -2% for the power coefficients, for pitch angles of 25° and 35°.

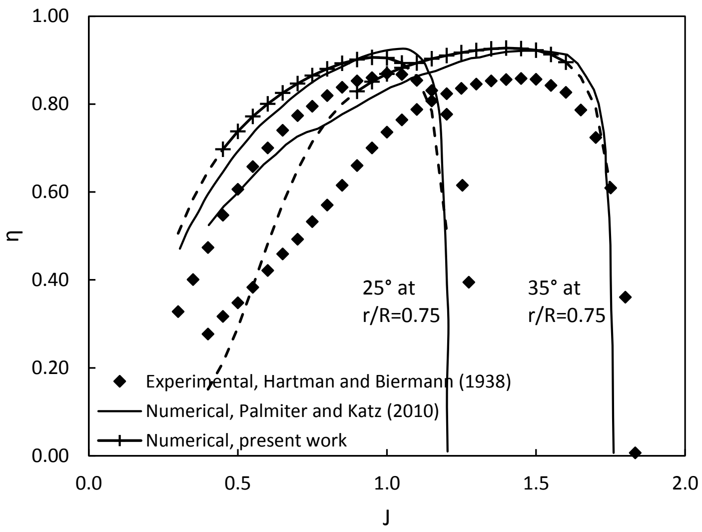

Fig. 5 Variation of the efficiency with the advance ratio for propeller Clark Y 5868-9

Table 1 Aerodynamic characteristics of the propeller Clark Y 5868-9 with two blades, diameter 3.048 m and pitch of 25° and 35° at 0.75R

pitch N, rpm		25° 1000			35° 800	
	Numerical (present)	Experimental	Error [%]	Numerical (present)	Experimental	Error [%]
J	0.950	0.950	---	1.400	1.400	---
η	0.906	0.856	5.4	0.927	0.856	8.4
k_T	0.049	0.052	-7.0	0.064	0.060	6.9
k_P	0.051	0.058	-11.8	0.096	0.098	-1.9
T, N	1434	1542	-7.0	1198	1120	6.9
Q, Nm	729	827	-11.8	877	894	-1.9
P, W	76358	86630	-11.8	73495	74910	-1.9

Both the panel method due to Palmiter and Katz (2010) and the present method allow calculate the loading along blade as can be seen in Fig. 6 for the case of pitch angle of 25°. As can be seen the loading curves follow the same tendencies but the present method shows higher values from about 60% of the rotor radius and also higher values at the blade tip. Perhaps this is due to the fact the panel method represent better the local geometry of the airfoil section than the present method. They reported that a big part of the blade showed inadequate pressure distribution calculated by the panel method. Hence, they suggested modifying the original blade geometry of Clark Y 5868-9 to obtain better flow conditions along the blade.

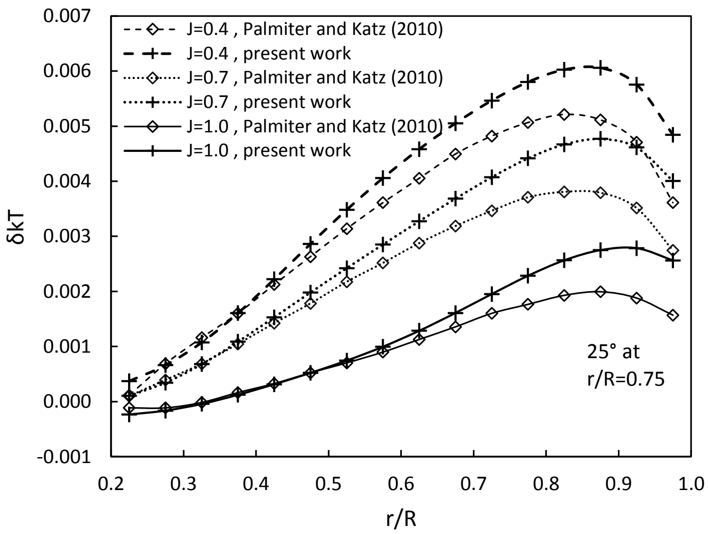

Fig. 6 Comparison between the present predictions of the radial loading along the blade and Palmiter´s and Katz results

4. RESULTS AND DISCUSSION

Hartman and Biermann (1938) presented the experimental results of the propeller Clark-Y 5868-9, having Clark-Y as an airfoil for the blades, it has two blades, 10 ft diameter and pitch of 25^{o}. The data for the Clark-Y(B) is available in Lyon *et al.* (1998) for Reynolds number of 3.0×10^{5}. This propeller is referred to in this paper as the reference propeller.

The validated code was used to calculate a propulsor with a new airfoil Göttingen 796 similar to the original airfoil Clark-Y but has a higher value of the ratio Cl/Cd. The characteristics of Gottingen 796 were determined from XFOIL for Reynolds numbers of 0.50×10^{6}, 0.75×10^{6}, 1.00×10^{6}, 1.25×10^{6}, 1.50×10^{6}, 1.75×10^{6} e 2.00×10^{6}. The curves of the lift and drag coefficients were obtained in terms of the angle of attack for increments of 0.25°, and incorporated in the numerical code.

Propeller 1 has exactly the same geometry and operational conditions as the reference propeller except that the airfoil section is Gottingen 796 instead of Clark-Y(B).

Propeller 2 is exactly as propeller 1 except that it has three different airfoil sections along the blade, where airfoil Göttingen 449 is used for the root region, airfoil Göttingen 796 for the middle part of the blade and airfoil Göttingen 622 for the tip region of the blade. The root region is defined between r/R=20% and r/R=40%, while the tip region is defined between r/R= 80% and the blade tip.

Fig. 7 shows the predicted results of the thrust, power and torque coefficients as well as the efficiency for propeller 1. The general tendency of the predictions is as expected.

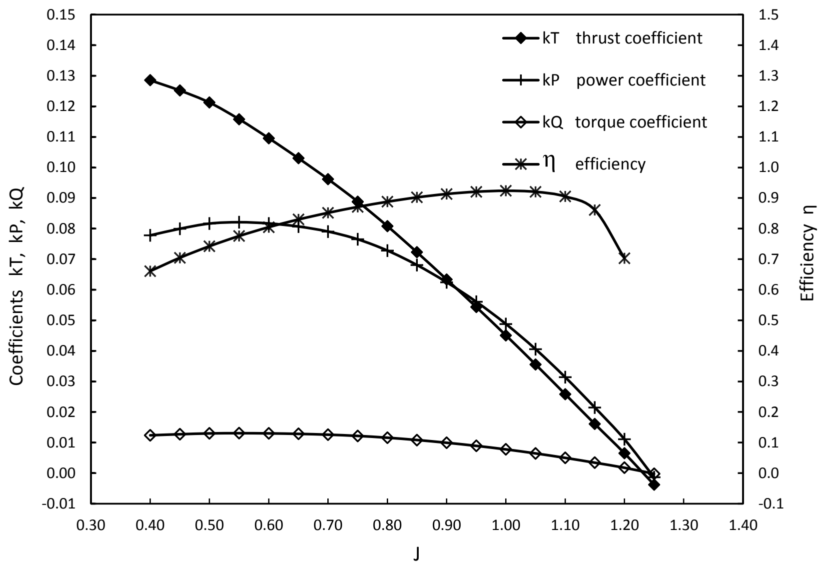

Fig. 7 Variation of the thrust, power and torque coefficients as well as the efficiency with the advance ratio for Propeller 1.

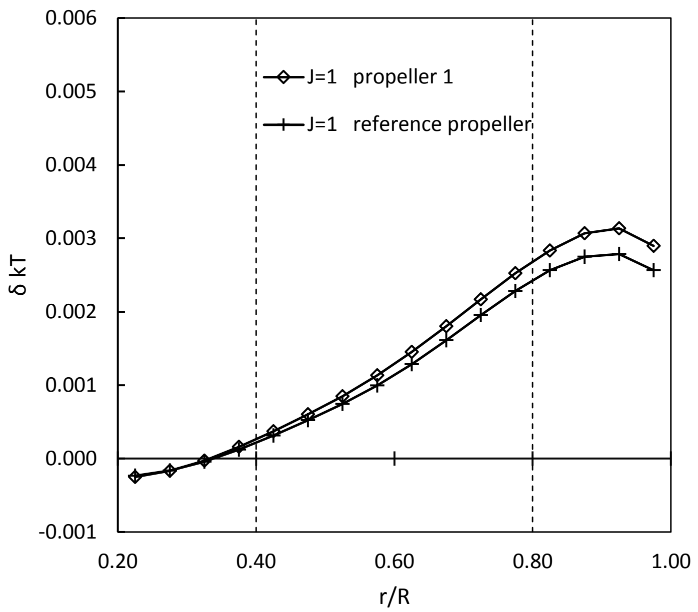

Fig. 8 Comparison of radial distribution of propeller 1 and reference propeller

Fig. 8 shows a comparison between propeller 1 and the reference propeller for the advance ratio J=1 corresponding to maximum efficiency. As can be seen, the loading of propeller 1 is more than that of the reference propeller in the middle and tip regions due to the fact that the Göttingen 796 airfoil has $(Cl/Cd)_{\max}$ more than that of the Clark-Y airfoil. The respective contributions of the three regions of propeller 1 to the total thrust are: -1% for root region, 48% for middle region and 53% for the tip region.

One can also observe that the tip loading is big for both propellers. In order to uniformize the blade loading and reduce its distribution at the tip we used three different profiles along the blade as in propeller 2. The airfoil Göttingen 622 was chosen for the tip region because it is thin and has a small camber while the Göttingen 449 was chosen for the root section because it is thick and hence adequate for the mechanical fixation of the blades. The Göttingen 796 airfoil of 3.5% camber was chosen for the middle region.

Fig. 9 shows the thrust loading coefficient for the propeller 1 with one airfoil along the blade and propeller 2 with three airfoils along the blade. As can be seen propeller 2 shows bigger loading at the root region and lower loading at the tip region. This is due to the fact that the Göttingen 449 airfoil at the root region has a camber ratio of 5.4% more than that of the Göttingen 796 of 3.5% camber of propeller 1 in the same region. The tip region Göttingen 622 airfoil has a camber ratio of 2.4% that less than that for case of propeller 1.

The respective contributions of the three regions of propeller 2 to the total thrust are 2.2 % for root region, 51.4% for middle region and 46.4% for the tip region.

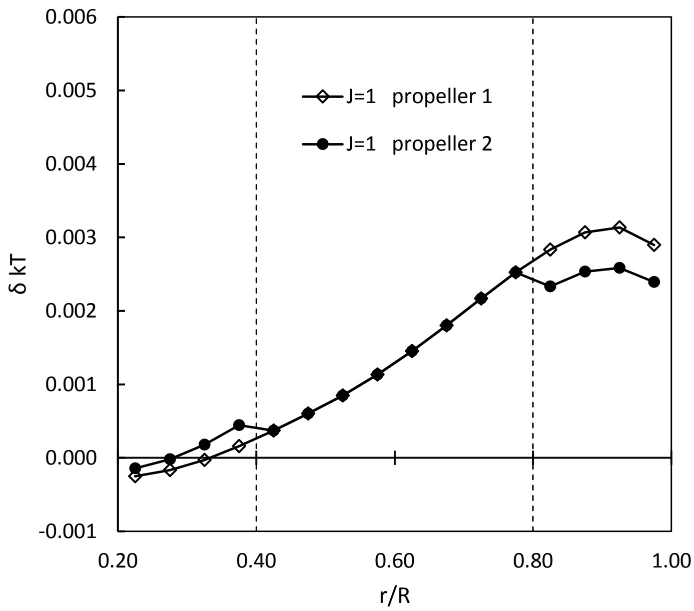

Fig. 9 Comparison of radial distribution of propeller 1 and propeller 2

Figs. 10-12 show the performance coefficients for the two propellers. One can observe that the thrust and power coefficients of propeller 2 are less than those of propeller 1. The efficiency curve seems to less affected by using different airfoils along the blade length. One can also observe that the range of the advance ratio J is less, varying from 0.55 to 1.2.

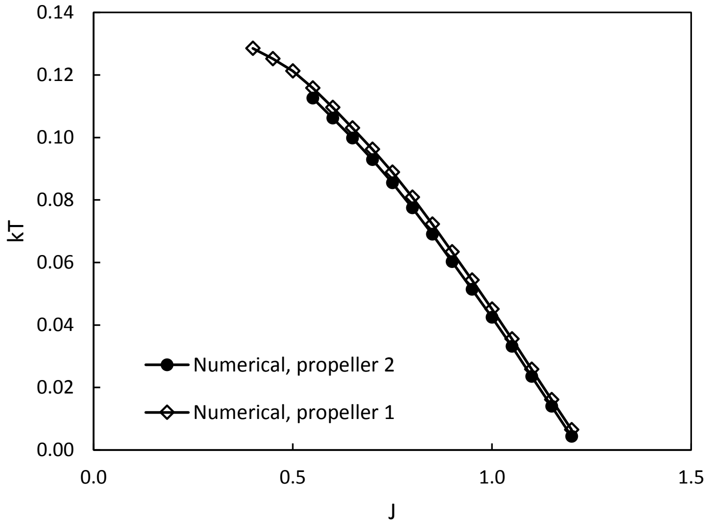

Fig. 10 Comparison of the thrust coefficients for propellers 1 and 2

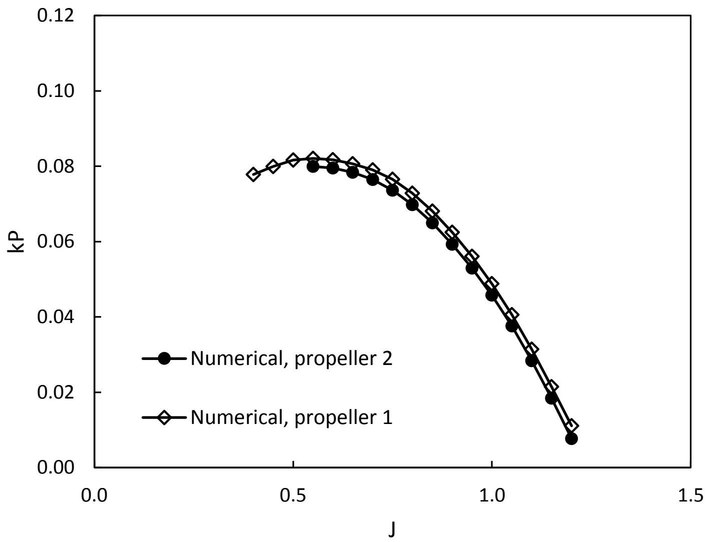

Fig. 11 Comparison of the power coefficients for propellers 1 and 2

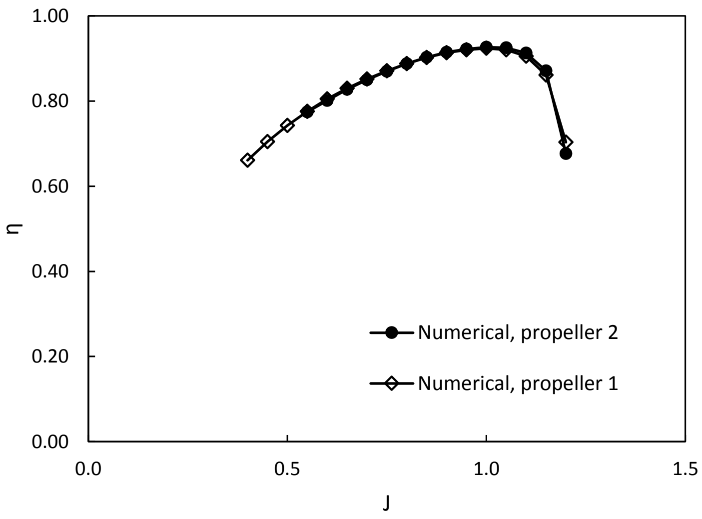

Fig. 12 Comparison of the efficiency for propellers 1 and 2

5. CONCLUSIONS

The proposed model is adequate for predicting the performance of small propellers with acceptable accuracy and small computing time.

XFOIL and XFOIL tools can be used to predict the aerodynamic characteristics and incorporate these data in the model to predict the performance of the same rotor but with different airfoil. Using different airfoils along the blade can be used to have a uniform loading distribution along the blade as required.

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