

Comparative Analysis of Vibration Isolator with Quasi-zero Stiffness

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Abstract

In recent years, vibration isolation with quasi-zero stiffness has become an emerging research area. Great amount of efforts have been made by researchers and engineers since its property of high-static-low-dynamic stiffness. It shows significant superiority on isolating vibration and supporting sufficient load simultaneously, especially under low and ultra-low frequency excitation. The current state-of-the-art literature study on quasi-zero stiffness vibration isolator (QSVI) is presented in this paper. The existing devices are classified into four types based on the negative stiffness elements, including spring, pre-buckled beam, magnet and nonlinear structure. The concept, characteristics and realisation of these QSVIs are summarized. In addition, the paper places its special interest in the comparative analysis of the properties of four representative QSVIs, including effective stiffness region, amplitude-frequency response and vibration isolation effect under force excitation. Finally, the evaluation results and general advice are proposed.

Keywords: quasi-zero stiffness, vibration isolator, effective stiffness region, force transmissibility, dynamic modelling

1 Introduction

As an effective vibration protection method of engineering structures, vibration isolation technique has been widely investigated and used in engineering applications. The principle of vibration isolator is to decouple a superstructure from vibration source by inserting a soft isolation layer between the structure and the source excitation. When the ratio of excitation frequency and natural frequency of system is larger than $\sqrt{2}$, the transmissibility is smaller than 1 indicating good vibration isolation effect. However, for an elastic linear vibration isolator, better vibration isolation effect requires lower stiffness to generate desirable low transmissibility, which means large deflection and weak load carrying capacity. Quasi-zero stiffness vibration isolator (QSVI) with nonlinear force and displacement characteristic is one of the most promising solutions for addressing this contradictory. Since it can realize significant vibration isolation effect

with low dynamic stiffness, in the meantime maintain sufficient load carrying capacity with high static stiffness.

The core elements for a QSVI is the negative stiffness elements. In 1957, Molyneux¹ submitted a research report to Aeronautic Research Council in the UK. In his pioneer work, he firstly proposed the negative stiffness concept and designed a device with extremely low stiffness. The device was initially composed of two oblique springs as energy storage elements to generate negative stiffness. However, he found the device cannot support the upper subject with large deflection, which was unstable. Therefore, in his later designs, he connected negative stiffness elements with a vertical spring to provide positive stiffness and adjusted the stiffness of this device to be quasi-zero stiffness, so-called QSVI. The combination contributes to a nonlinear force-displacement relationship, which can realize desirable vibration isolation effect without sacrificing the load carrying capacity. After that, several modified QSVIs have been proposed and investigated by researchers.

Since negative stiffness elements are unstable and cannot work by themselves, they need to work with positive stiffness elements (normally in form of mechanical springs) to form QSVI system. According to the types of the commonly utilized negative stiffness components, there are three kinds of QSVIs, which are QSVIs using energy storage elements (spring and pre-buckled beam etc.), QSVIs using magnetic elements (permanent magnets, electromagnets or hybrid magnets etc.) and QSVIs using geometrically nonlinear structure (cam and convex surface etc.). Among that, the QSVIs using energy storage elements can be further classified into QSVIs using springs and QSVIs using pre-buckled beams.

In this paper, the current state-of-the-art literature study on the existing QSVIs will be presented based on the types of negative stiffness elements. A comparative analysis among four representative devices is conducted to evaluate their both static and dynamic performances.

2 Classification of QSVIs

2.2 QSVIs using springs

The core elements of QSVIs using springs are the pre-compressed springs that provide restoring force component to assist deformation to generate negative stiffness. As aforementioned, through adjusting the stiffness of oblique and vertical springs, the total stiffness of a QSVI could be zero at the equilibrium position. The two oblique springs in Molyneux's device¹ do not influence the load carrying capacity at equilibrium position but provide negative stiffness in the vertical direction²⁻⁴. Built on this design, Carrella conducted comprehensive research on the static and dynamic properties of this model and proposed the maximum excursion as an evaluation index to optimize the stiffness ratio. After that, he further modified the original device and designed a novel QSVI⁴⁻⁶.

Beside coil spring, other springs and spring structures can also be adopted to provide restoring force. Lan and Yang^{7,8} replaced the coil springs with special planar springs to design a compact QSVI. Niu and Meng⁹ designed a QSVI with one disk spring instead of two oblique coil springs, which could restore and produce more axial

nonlinear force with small displacement at equilibrium position than Molyneux's device ¹. Besides, a novel QSVI with horizontal-spring structure was proposed, which has been applied to the vibration isolation of vehicle seat ¹⁰⁻¹². In Fig. 1, it was designed with two symmetrical sliding block-bar structures installed in the horizontal direction. The two oblique springs only move in the horizontal direction by utilizing the sliding block-bar structures, which can improve the stability and accuracy of this device. The horizontal-spring QSVI designed by Antoniadis ¹³ is asymmetric to the device shown in Fig. 1. Two additional vertical springs are added to improve the load carrying capacity in this design. Besides, considering that in practice it is difficult to keep the stiffness zero at equilibrium position, an engineering safety margin was introduced to conduct the parametric analysis to optimize the device.

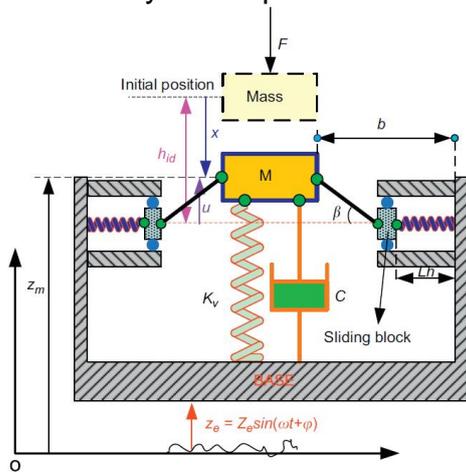


Fig. 1. Schematic representation of horizontal-spring passive QSVI for vehicle seat ¹⁰⁻¹²

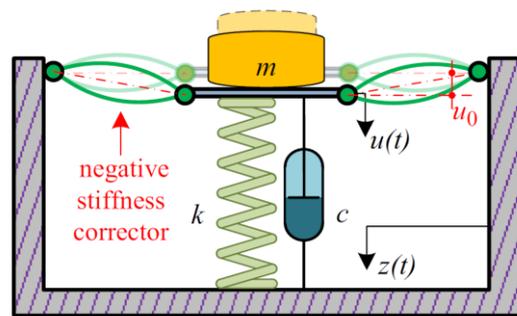


Fig. 2. Schematic representation of a hinged joint QSVI ¹⁴

2.2 QSVIs using pre-buckled beams

When the applied load exceeds the buckling load of an elastic column, it will be buckled with initial imperfection and restored energy, which can be used as negative stiffness element. Pre-buckled beams with negative stiffness can be used to develop QSVIs with coil springs of positive stiffness. Liu et al. designed a QSVI using pre-buckled beams that can be regarded as an evolving three-spring QSVI. The vertical force of this device is composed of two parts, including the spring force and the force provided by the buckled-beams. In practice, due to the imperfections in fabrication and installation as well as the changing environment, the device cannot precisely maintain stable at the horizontal equilibrium position where the dynamic stiffness should be equal to zero. Hence, Liu et al. investigated the influence of imperfection on the performance of QSVI. Similarly, Huang ¹⁴ adopted two-hinged joint thin-walled beams with initial imperfection to design a novel QSVI, which is illustrated in Fig. 2. By adjusting the values of structural parameters, the device can realize zero stiffness at equilibrium position. They verified that both stiffness and load imperfection would increase the total stiffness of the device, hence leading to unwanted high natural frequency.

Besides, the vertical mechanical coil spring can also be replaced by buckled beams. Winterflood¹⁵ established a high performance vibration isolator by employing two Euler buckling springs to replace the vertical coil springs. Zhang¹⁶ used a beam under axial force at its two ends to get the same kind of high-static-low-dynamic stiffness properties. Kashdan¹⁷ and Fulcher¹⁸ designed three different kinds of meso-scale QSVI devices, including single-beam, uncoupled double-beam and coupled double-beam configurations with first-mode and third-mode buckling supports separately. Mori¹⁹ demonstrates the advantage of utilizing buckled beams as negative stiffness elements is that it can further reduce the ratio of local stiffness and static stiffness compared with a linear mechanical spring.

2.3 QSVIs using magnetism

The QSVIs using magnetism is based on the principle that when the magnet leaves the equilibrium position, a magnetic force difference will be generated in the same direction of relative displacement. The magnetic force assisting relative movement is inversely proportional to the square of the distance²⁰. The magnetic force can be provided by permanent magnets, electromagnets and hybrid magnets, which are normally used as magnetic springs to provide negative stiffness. A couple of magnetic springs can work solely or cooperate with other positive stiffness elements, e.g. coil springs, to construct a QSVI using magnetism.

Robertson²¹ conducted an analysis of a magnetic levitation device composed of a pair of fixed cuboid magnets in vertical direction and a magnetic-sensitive mass suspended between two cuboid magnets. Similarly, Wu²² also developed a magnetic QSVI using tri-magnets and conducted experiments to verify its ability in reducing the inherent frequency of vibration isolation device without sacrificing its load supporting capacity. Mizuno²³ optimized the combination of permanent magnets and coil springs to achieve high-static-low-dynamic stiffness. A QSVI composed of linear mechanical springs, smooth bar and magnets was designed, the stability and the set-up of which were significantly improved³.

Shan²⁴ designed a miniaturized QSVI with two ring magnets and pneumatic, that can reduce the resonance frequency from 3.61 to 2.34 Hz through adjusting the stiffness values of pneumatic spring and magnetic spring. In Fig. 3, a QSVI was designed with mechanical spring with positive stiffness and two ring magnets to provide negative stiffness²⁵. The properties of this device vary greatly with the height difference between the two ring magnets but change slightly when the difference keeps constant. Besides, there are several QSVIs adopt electromagnets to replace permanent magnets. For example, Mizuno²⁶, Zhou²⁷, Easu²⁸ utilized hybrid magnets consisting permanent magnets and electromagnets to build semi-active magnetic QSVI. Compared with the permanent-magnet system, the hybrid-magnet system are more stable and can realize controllable properties.

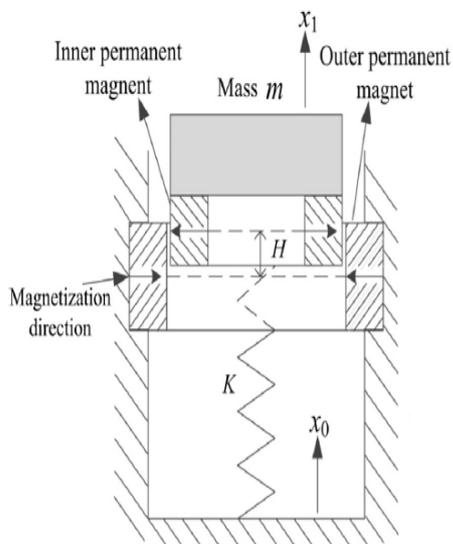


Fig. 3. Schematic representation of a ring-magnet QSVI ²⁵

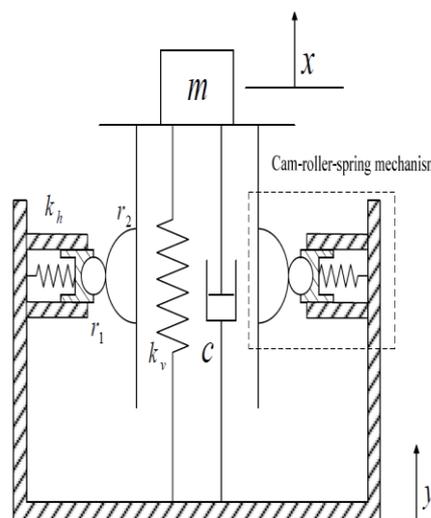


Fig. 4. Schematic representation of a convex QSVI ^{29, 30}

2.4 QSVIs using geometrically nonlinear structures

The principle of geometrically nonlinear structures in generating negative stiffness is attributed to the continuously changing acceleration when the mass slides along the curved surface due to the existing of gravity. Originally, the mass can only move along horizontal direction since the dependence of gravity, which is not suitable for vertical vibration isolation. The QSVI proposed by Zhou ²⁹ and Cheng ³⁰ with high-static-low-dynamic stiffness breaks the direction limitation. The device composed of rollers, cams, and springs etc. is depicted in Fig. 4. There are two states during the working process of this device, including contact state and detached state, which makes it suitable for both smaller and larger amplitude excitation. This QSVI can achieve negative stiffness at its static equilibrium position and the stiffness changes in a steady way. In addition, the wider the working region is, the higher the negative stiffness degree is. Following that, Zhou ³¹ subsequently proposed a novel six-degree-of-freedom vibration isolation platform, which can mitigate multiple-direction vibration in a broad bandwidth with high effectiveness under low-frequency range.

3 Comparative analysis of representative vertical QSVIs

As aforementioned, QSVIs used for vibration isolation in vertical direction can be categorized into four groups based on the core negative stiffness elements. Although, there are a series of QSVIs have been designed and investigated, there is no systematic analysis of the comparative evaluation among them in the exiting research results. Hence, to analyse their features and evaluate their performances, in this section, a comparative analysis between four different representative QSVIs was completed, including the analysis of stiffness properties and numerical modelling of vibration isolation effect. The four representative devices are listed in Table 1.

Table 1. Information of representative devices

No	Fig No.	Type	Core elements	Inventers
1	Fig. 1	Horizontal-spring device for vehicle seat	Linear spring+ sliding block-bar structures	Le et al., 2011, 2013a, 2013b
2	Fig. 2	Double hinged joint thin-walled beams device	Double hinged joint thin-walled beams+ linear spring	Huang et al., 2014
3	Fig. 3	Spring-magnet device	Magnets+ pendulum springs	Shan et al., 2015
4	Fig. 4	Convex device	Cams+ rollers+ linear springs	Cheng et al., 2016

The main purpose of this comparative analysis is to obtain insight of the QSVIs designed with different mechanisms and to understand the pros and cons of each type of device. To evaluate their performances on a fair basis, the following assumptions are proposed:

- (1) The key evaluative indexes are the non-dimensional stiffness and transmissibility;
- (2) The initial position of four devices are set at equilibrium position where the non-dimensional stiffness equals to zero;
- (3) Four devices are equipped with same level of damping.
- (4) The non-dimensional stiffness of four devices is set to one at a same specific position.

3.1 Static analysis

The expressions of dimensionless force and stiffness of these four representative devices are derived and listed below. The curves of dimensionless stiffness vs displacement are shown in Fig. 5.

$$\text{No. 1} \quad \begin{cases} \hat{F} = \hat{h}_{id} - \frac{1}{\sqrt{\gamma_1^2 - \hat{x}^2}} - \frac{\gamma_2}{\sqrt{\gamma_1^2 - \hat{x}^2}} + 1)x \\ \hat{K} = 1 + 2\alpha \left(\frac{\hat{x}^2(\gamma_2 - 1)}{(\gamma_1^2 - \hat{x}^2)^{3/2}} - \frac{(1 - \gamma_2) + \sqrt{\gamma_1^2 - \hat{x}^2}}{\sqrt{\gamma_1^2 - \hat{x}^2}} \right) \end{cases} \quad (1)$$

$$\text{No. 2} \quad \begin{cases} \hat{F} = (1 - k_1\lambda) - k_3\lambda x^3 + \sqrt{1 - \gamma^2} \\ \hat{K} = 1 - k_1\lambda + 3k_3\lambda \hat{x}^2 \end{cases} \quad (2)$$

$$\text{No. 3} \quad \begin{cases} \hat{F} \approx \hat{x} - \frac{\beta \hat{x}}{(\alpha + \hat{x}^2)^2} \\ \hat{K} \approx 1 - \frac{\beta(3\hat{x}^2 - \alpha)}{(\alpha + \hat{x}^2)^3} \end{cases} \quad (3)$$

$$\text{No. 4} \quad \begin{cases} \hat{F} = \begin{cases} (1 - 2\beta(1 + \frac{l-1}{\sqrt{1-\hat{x}^2}}))\hat{x} \\ \hat{x} \end{cases} & (|\hat{x}| \leq x_d) \\ \hat{K} = \begin{cases} 1 - 2\beta(1 + \frac{l-1}{(1-\hat{x}^2)^{3/2}}) \\ 1 \end{cases} & (|\hat{x}| > x_d) \end{cases} \quad (4)$$

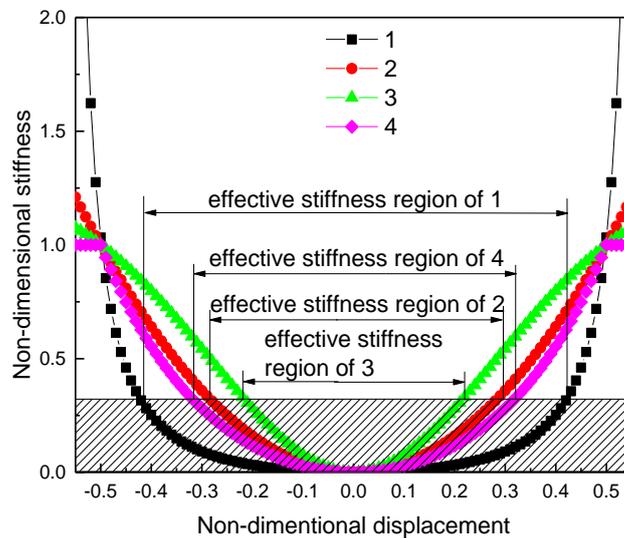


Fig. 5. The relationships of non-dimensional stiffness and non-dimensional displacement of four representative devices

The curves of non-dimensional stiffness vs displacement of these four devices are depicted in Fig. 5. For comparison reason, all the curves are set to cross at one point by adjusting the structure parameters of the devices. A threshold stiffness K_{lim} is defined to evaluate the non-dimensional stiffness and only when it is smaller than 1, the total stiffness of this QSVI is smaller than the stiffness of vertical spring. The displacement range around equilibrium position that makes stiffness smaller than the threshold stiffness K_{lim} is called effective stiffness region. In practice, a desirable QSVI should be designed with wide effective stiffness region, i.e. the QSVI can maintain low stiffness during a large displacement stroke. Fig. 5 shows that the effective stiffness region of device 1 is significant wider than that of other devices. Besides, its stiffness changes smoothly during the effective stiffness region, while increases dramatically outside this region. The non-dimensional stiffness curves of device 2, 3 and 4 all increase gradually without obvious flat and steep region (effective stiffness region of

device 4 > device 2 > device 3). In addition, there is a threshold stiffness value for device 4 due to its design combined with working states. Therefore, from the view of effective stiffness region, QSVI 1 is the most desirable among four representative devices and following that is device 4.

3.2 Dynamic analysis

Compared with linear vibration isolator, one of the typical advantages of QSVI is the extremely low natural frequency that contributes to larger frequency ratio and then lower transmissibility. Hence, it can realize superior vibration isolation effect and avoid the happening of resonance, especially under low and ultra-low frequency excitation. The vibration isolation effects of these four different devices were simulated to evaluate its dynamic properties. The single-degree-of-freedom mass-spring-damper systems with the same mass, damping ratio, and force excitation shown in Fig. 6 were established in MATLAB/Simulink. The governing equation of these systems is described by equation (5). A sweep frequency force excitation and a sinusoidal force excitation were imposed on the mass to generate vibration in vertical direction respectively. The force transmitted to the base after being isolated by QSVIs can be used to reflect the transmissibility of vibration isolator when all these four devices are under same excitation force. The frequency responses and time-history responses of them are described in Fig. 7 and Fig. 8.

$$m\ddot{x} + c\dot{x} + k(x)x - mg = F_0 \cos(\Omega t) \quad (5)$$

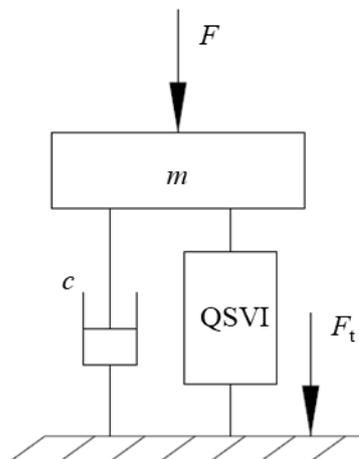


Fig. 6. The schematic of modelling device under force excitation

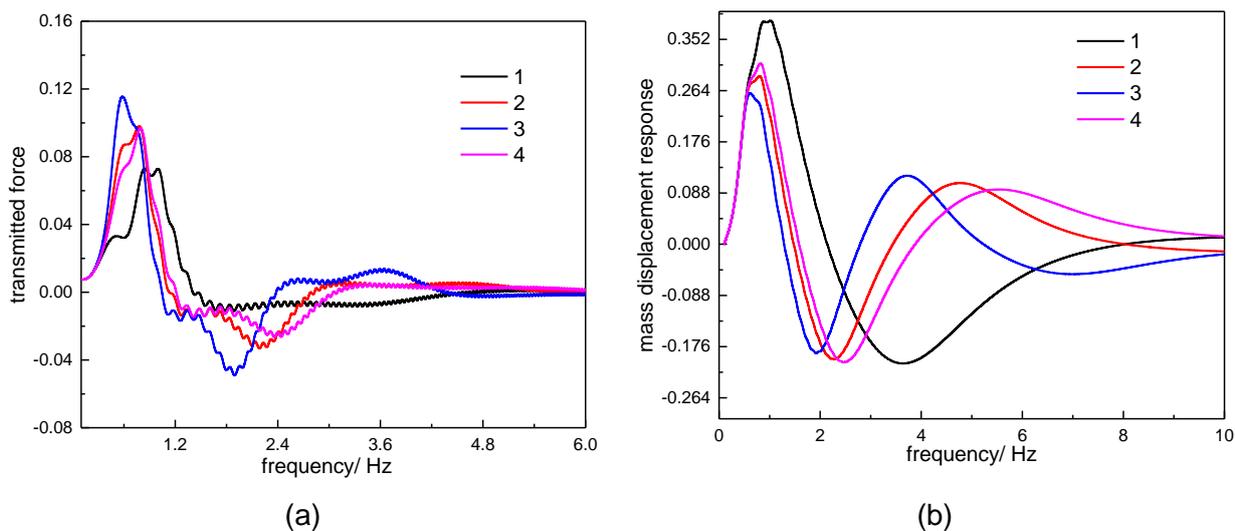


Fig. 7. The effect of these four devices under sweep sinusoidal force excitation (a) transmitted force, (b) displacement response

For the dynamic analysis, a sweep frequency sinusoidal signal with constant amplitude is input to these four QSVIs. The frequency range of this excitation is mainly spread within 0.1-10 Hz. The frequency responses of these four devices, including transmitted force and displacement response are presented in Fig. 7, which tells the frequency where the QSVIs will resonate. Among them, device 1 shows significant better effect on reducing force transmissibility and the performance of device 3 is relatively poor due to its stiffness is the largest under the effective stiffness region. However, the displacement response shows the opposite result that device 1 leads to larger deflection because of its lower stiffness. Besides, there are peak values on the transmitted force and displacement curves of the four devices. The peak values of transmitted force and corresponding frequencies are 0.073 at 1.0 Hz (device 1), 0.098 at 0.8 Hz (device 2), 0.116 at 0.6 Hz (device 3), and 0.097 at 0.8 Hz (device 4). The peak values of displacement response and corresponding frequencies are 0.38 (device 1), 0.29 (device 2), 0.26 (device 3), and 0.31 (device 4) respectively. The response frequency of device 3 is relatively small, which means it could work under a wider frequency region than the other three devices. Besides, there is no significant difference between the performance of device 2 and 4.

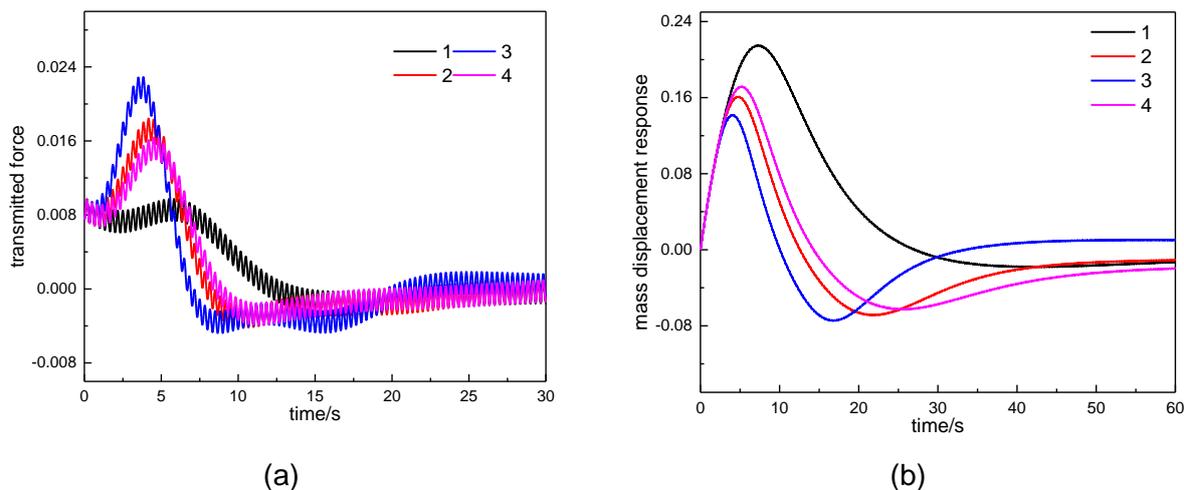


Fig. 8. The time-history response under sinusoidal force excitation $F = 0.15 \cos(3t)$
 (a) transmitted force, (b) displacement response

Since the QSVIs are designed with nonlinear force-displacement characteristic, that means variable stiffness, their natural frequencies will vary as the response goes on. Therefore, more numerical simulations are carried out for the four QSVIs to obtain their time-history responses to a same specified excitation. The excitation frequency is set larger than the natural frequency of the device with the vertical spring working alone to avoid the resonance to happen. The time-history responses of the representative devices under sinusoidal excitation is shown in Fig. 8. The time-history transmitted force curves indicate that device 1 can significantly reduce the force transmitted from the mass to the base, since the values of transmitted force is always smaller than the amplitude of the excitation. Although the transmitted forces are magnified at the beginning stage for device 2, 3 and 4, the forces attenuate to zero as the response goes on rapidly. In Fig. 8 (b), similar to frequency response, the displacement response of device 3 is the smallest, hence avoiding large deflection. Besides, all displacement responses are less than 0.3, which means the devices are working under effective stiffness region with extremely low stiffness and small force transmissibility.

4 Conclusion

This paper conducted a comprehensive survey on the state-of-the-art research and development in utilising QSVIs for the vibration isolation of engineering structures. In order to make comparison among devices with different core elements, this paper attempted to evaluate the effective stiffness region, transmitted force and displacement response of four representative QSVIs. For comprehensively evaluating, it is difficult to determine which is the best, since under different situation, the requirements are different. Basically, device 1, the horizontal-spring device, is with better properties among all these four devices, since it can isolate most of the excitation force. However, if the most important requirement is low resonance frequency and small amplitude response, then the device 3 is better. There is no significant difference between the

performance of device 2 and 4, except there are two working stages for device 4, which can avoid the stiffness increasing dramatically outside the effective stiffness region.

References

- [1] Molyneaux, W., *Supports for Vibration Isolation*, ARC/CP-322. 1957.
- [2] Hao, Z. and Q. Cao, *The isolation characteristics of an archetypal dynamical model with stable-quasi-zero-stiffness*. Journal of Sound and Vibration, 2015. **340**: p. 61-79.
- [3] Carrella, A., et al., *On the design of a high-static–low-dynamic stiffness isolator using linear mechanical springs and magnets*. Journal of Sound and Vibration, 2008. **315**(3): p. 712-720.
- [4] Carrella, A., M.J. Brennan, and T.P. Waters, *Static analysis of a passive vibration isolator with quasi-zero-stiffness characteristic*. Journal of Sound and Vibration, 2007. **301**(3-5): p. 678-689.
- [5] Carrella, A., M. Brennan, and T. Waters, *Optimization of a quasi-zero-stiffness isolator*. Journal of Mechanical Science and Technology, 2007. **21**(6): p. 946.
- [6] Carrella, A., et al., *Force and displacement transmissibility of a nonlinear isolator with high-static-low-dynamic-stiffness*. International Journal of Mechanical Sciences, 2012. **55**(1): p. 22-29.
- [7] Lan, C.-C., S.-A. Yang, and Y.-S. Wu, *Design and experiment of a compact quasi-zero-stiffness isolator capable of a wide range of loads*. Journal of Sound and Vibration, 2014. **333**(20): p. 4843-4858.
- [8] Wu, T.-H. and C.-C. Lan, *A wide-range variable stiffness mechanism for semi-active vibration systems*. Journal of Sound and Vibration, 2016. **363**: p. 18-32.
- [9] Niu, F., et al., *Design and analysis of a quasi-zero stiffness isolator using a slotted conical disk spring as negative stiffness structure*. Journal of Vibroengineering, 2014. **16**(4).
- [10] Le, T.D. and K.K. Ahn, *A vibration isolation system in low frequency excitation region using negative stiffness structure for vehicle seat*. Journal of Sound and Vibration, 2011. **330**(26): p. 6311-6335.
- [11] Le, T.D. and K.K. Ahn, *Fuzzy sliding mode controller of a pneumatic active isolating system using negative stiffness structure*. Journal of Mechanical Science and Technology, 2013. **26**(12): p. 3873-3884.
- [12] Le, T.D. and K.K. Ahn, *Experimental investigation of a vibration isolation system using negative stiffness structure*. International Journal of Mechanical Sciences, 2013. **70**: p. 99-112.
- [13] Antoniadis, I., et al., *Hyper-damping properties of a stiff and stable linear oscillator with a negative stiffness element*. Journal of Sound and Vibration, 2015. **346**: p. 37-52.
- [14] Huang, X., et al., *Vibration isolation characteristics of a nonlinear isolator using Euler buckled beam as negative stiffness corrector: A theoretical and experimental study*. Journal of Sound and Vibration, 2014. **333**(4): p. 1132-1148.
- [15] Winterflood, J., D.G. Blair, and B. Slagmolen, *High performance vibration isolation using springs in Euler column buckling mode*. Physics Letters A, 2002. **300**(2-3): p. 122-130.

- [16] Zhang, J.-z., et al., *Study on ultra-low frequency parallel connection isolator used for precision instruments*. Zhongguo Jixie Gongcheng/China Mechanical Engineering, 2004. **15**(1): p. 69-71.
- [17] Kashdan, L., et al., *Design, fabrication, and evaluation of negative stiffness elements using SLS*. Rapid Prototyping Journal, 2012. **18**(3): p. 194-200.
- [18] Fulcher, B.A., et al., *Analytical and Experimental Investigation of Buckled Beams as Negative Stiffness Elements for Passive Vibration and Shock Isolation Systems*. Journal of Vibration and Acoustics, 2014. **136**(3).
- [19] Mori, H., et al. *The effect of beam inclination on the performance of a passive vibration isolator using buckled beams*. in *Journal of Physics: Conference Series*. 2016. IOP Publishing.
- [20] Akoun, G. and J.-P. Yonnet, *3D analytical calculation of the forces exerted between two cuboidal magnets*. IEEE Transactions on magnetics, 1984. **20**(5): p. 1962-1964.
- [21] Robertson, W.S., et al., *Theoretical design parameters for a quasi-zero stiffness magnetic spring for vibration isolation*. Journal of Sound and Vibration, 2009. **326**(1-2): p. 88-103.
- [22] Wu, W., X. Chen, and Y. Shan, *Analysis and experiment of a vibration isolator using a novel magnetic spring with negative stiffness*. Journal of Sound and Vibration, 2014. **333**(13): p. 2958-2970.
- [23] Mizuno, T., et al., *Vibration isolation system combining zero-power magnetic suspension with springs*. Control Engineering Practice, 2007. **15**(2): p. 187-196.
- [24] Shan, Y., W. Wu, and X. Chen, *Design of a miniaturized pneumatic vibration isolator with high-static-low-dynamic stiffness*. Journal of Vibration and Acoustics, 2015. **137**(4): p. 045001.
- [25] Zheng, Y., et al., *Design and experiment of a high-static–low-dynamic stiffness isolator using a negative stiffness magnetic spring*. Journal of Sound and Vibration, 2016. **360**: p. 31-52.
- [26] Mizuno, T., T. Tsumiya, and M. Takasaki, *Vibration isolation system using negative stiffness*. JSME International Journal Series C Mechanical Systems, Machine Elements and Manufacturing, 2003. **46**(3): p. 807-812.
- [27] Zhou, N. and K. Liu, *A tunable high-static–low-dynamic stiffness vibration isolator*. Journal of Sound and Vibration, 2010. **329**(9): p. 1254-1273.
- [28] Easu, D. and A. Siddharthan, *Theoretical and Experimental Analysis of a Vibration Isolation System Using Hybrid Magnet*. Procedia Engineering, 2013. **64**: p. 1139-1146.
- [29] Zhou, J., et al., *Nonlinear dynamic characteristics of a quasi-zero stiffness vibration isolator with cam–roller–spring mechanisms*. Journal of Sound and Vibration, 2015. **346**: p. 53-69.
- [30] Cheng, C., et al., *On the analysis of a high-static-low-dynamic stiffness vibration isolator with time-delayed cubic displacement feedback*. Journal of Sound and Vibration, 2016. **378**: p. 76-91.
- [31] Zhou, J., et al., *A novel quasi-zero-stiffness strut and its applications in six-degree-of-freedom vibration isolation platform*. Journal of Sound and Vibration, 2017. **394**: p. 59-74.