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A density correction method for smoothed particle hydrodynamics

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ABSTRACT

Smoothed particle hydrodynamics is one of the widely used particle methods. The SPH has attractive features to simulate large deformation problems because an analysis domain is discretized by a finite number of particles. However, numerical drawbacks exist in SPH. Particle inconsistency is a major issue which deteriorates SPH solutions. Especially, unphysical density and pressure oscillations occur due to the particle inconsistency. In this presentation, we introduce a new density correction method recently developed to resolve such phenomenon. Through the proposed density correction method, fluid pressure is reliably evaluated without density dissipation problems [1]. Furthermore, the proposed density correction method can be applied to various engineering applications including dynamics of floating structures. The performance of the proposed method is demonstrated by various numerical examples.

1. Introduction

Smoothed particle hydrodynamics (SPH) is one of the widely used particle methods. The SPH is originally invented for solving open space astronomy problems [2, 3]. After then, the SPH is applied to analyze free surface flows [4] and is used in various engineering fields [5-8]. In the SPH, a finite number of the particle is used to describe the motion of flows. The SPH has an attractive feature to deal with free surface or interface without complicated treatment.

A physical value is approximated through the information of near particles based on a kernel function. In the SPH, there are some numerical drawbacks that is should be

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resolved to obtain reliable solutions. One of the main issues is the particle inconsistency problem. The particle inconsistency causes numerical errors in the estimation of derivatives for physical values. The numerical errors induce the unphysical density variations that deteriorate the solutions. In the SPH, the two conditions should be satisfied: partition of unity and symmetric condition. However, the particle distributions and the number of interacting particles are continuously changed during simulation. The particle inconsistency is an inherent numerical problem. To obtain a reliable solution, the unphysical density variations induced by the particle inconsistency should be resolved.

In this paper, we introduce a new density correction method to resolve the unphysical density variations. Furthermore, the density correction method is applied to the dynamics of floating structures and its performance is demonstrated through various numerical examples.

2. SPH formulation

For the weakly compressible flows, the following governing equations are considered

$$\frac{D\rho}{Dt} = -\rho\nabla\cdot\mathbf{v}\,, \tag{1}$$

$$\frac{D\mathbf{v}}{Dt} = -\frac{1}{\rho}\nabla p + \frac{\mu}{\rho}\nabla^2\mathbf{v} + g\,, \tag{2}$$

where ρ is the density, $\mathbf{v}$ is the velocity, p is the pressure, μ is the dynamic viscosity, and g is the gravitational acceleration.

For the barotropic flows, the pressure p is calculated by the density ρ as follows

$$p = \frac{c_0^2\rho_0}{\gamma}[(\frac{\rho}{\rho_0})^\gamma - 1]\,, \tag{3}$$

in which c_0 is the reference sound speed, ρ_0 is the reference density, and γ is a constant value that is 7 in usual water.

In the SPH approximation, the governing equation is discretized by two procedures: integral representation and particle approximation. The discretized form of governing equation is described as

$$\frac{D\rho_i}{Dt} = \rho_i\sum_{j=1}^{N}\frac{m_j}{\rho_j}\mathbf{v}_{ij}\nabla_i W(\mathbf{x}_{ij}, h)\,, \tag{4}$$

$$\frac{D\mathbf{v}_i}{Dt} = \sum_{j=1}^{N} m_j \left(\frac{p_i + p_j}{\rho_i \rho_j} + \Pi_{ij}\right) \nabla_i W(\mathbf{x}_{ij}, h) + \frac{m_j(\mu_i + \mu_j)\mathbf{x}_{ij}\mathbf{v}_{ij}}{\rho_i \rho_j \mathbf{x}_{ij}^2} \nabla_i W(\mathbf{x}_{ij}, h) + g \ , \tag{5}$$

where N is the number of interacting particles, $\mathbf{v}_{ij}$ is the relative velocity between particles i and j, and $\nabla_i W$ is the gradient of a kernel function at particle i.

In Eq (4), The term Π_{ij} is artificial viscosity that makes the simulations stable.

$$\Pi_{ij} = -2\alpha hc \frac{(\mathbf{v}_{ij} \cdot \mathbf{x}_{ij})}{(\rho_i + \rho_j)(\mathbf{x}_{ij}^2 + (0.1h)^2)} \ , \tag{6}$$

in which α is a constant that controls the intensity of artificial viscosity.

3. Density correction

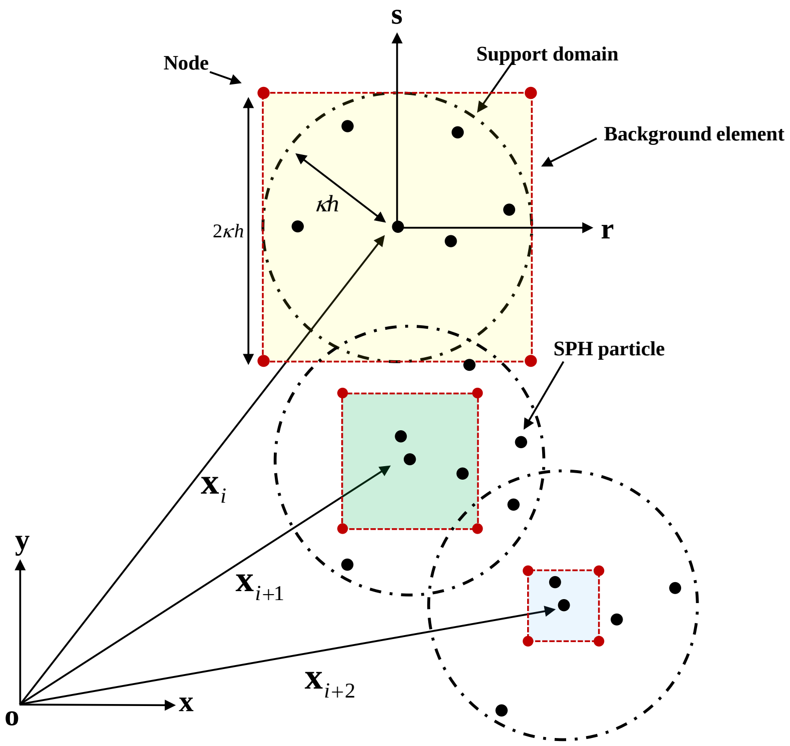

Figure 1. SPH particles with various background elements

A new density correction method is introduced to resolve the unphysical density oscillations and peaks. As shown in Fig. 1, the background element is employed and each particle moves freely with the background element. To define the density interpolation field, the nodal density values are estimated as follows, see Ref. [1]

$$D_j = \frac{1}{M_j} \sum_{k=1}^{N} H_j(r_k, s_k) m_k \rho_k \qquad \text{for } j = 1, 2, 3, 4 \tag{7}$$

with

$$M_j = \sum_{k=1}^{N} H_j(r_k, s_k) m_k \ , \tag{8}$$

in which N is the number of interacting particles, H_j is the shape function at the node, and (r_k, s_k) are the local coordinates of the particle. Using the nodal density values D_j, the density interpolation field for the particle i is defined as

$$\eta_i(r,s) = \sum_{j=1}^{4} H_j(r,s) D_j \ . \tag{9}$$

Finally, the corrected density value is obtained through the density interpolation field as follows $\bar{\rho}_i^{fluid} = \eta_i(0,0)$. (10)

4. Results

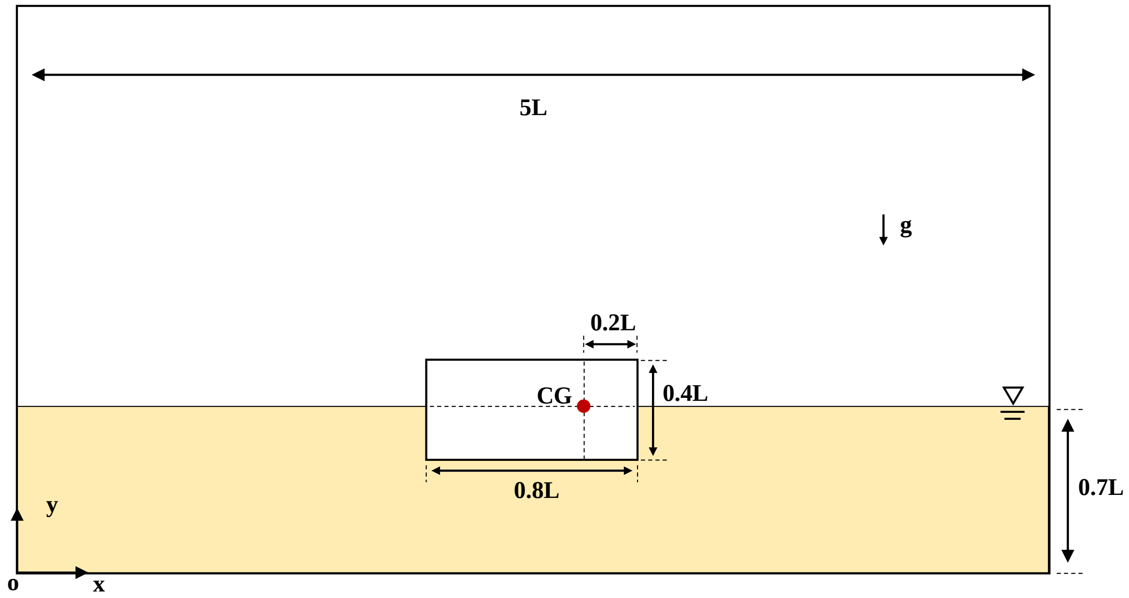

Figure 2. Configuration of a floating structure interacting with free surface flow

As shown in Fig. 2, a rectangular floating body is half-submerged in water. The center of gravity (CG) is located at a horizontally shifted point. The mass of the floating structure is $0.16\rho L^2$ and the moment of inertia $0.01072\rho L^4$. Through the proposed density correction method, the fluid pressure acting on the floating structure is evaluated and the numerical results are compared with the exact solution. Initially, the floating structure does not satisfy the hydrostatic equilibrium.

Fig. 3 shows the dimensionless vertical position of the center of gravity (CG). The results obtained by the proposed method exactly converge to the exact solution denoted by the black line in Fig. 3. It is clearly identified that reliable fluid pressure is obtained through the proposed method.

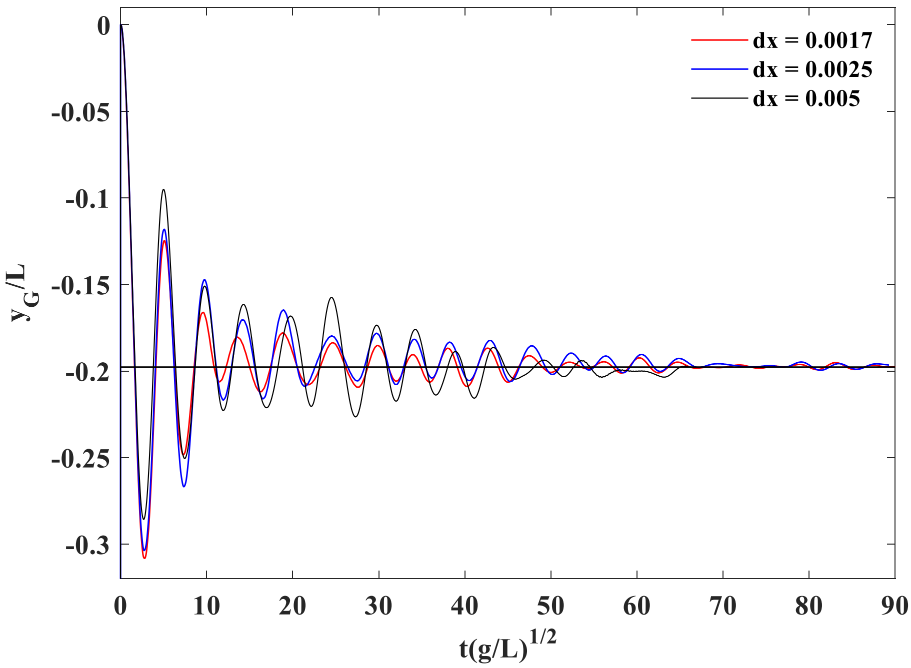

Figure 3. Dimensionless vertical position of center of gravity (CG).

5. Conclusions

In this paper, we introduced a new density correction method for smoothed particle hydrodynamics (SPH) and the density correction method was applied to engineering applications such as dynamics of floating structures. Through the proposed method, reliable fluid pressure was evaluated, resolving unphysical density variations. To demonstrate the performance of the proposed method, a numerical example was considered and the results were compared with the exact solutions. The results show that the proposed method reduces the unphysical density oscillations and peaks that induce the numerical errors and hydrodynamic loads are exactly evaluated.

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