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Bi-objective optimization of functionally graded beams in a thermal environment

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ABSTRACT

A mixed layer-wise (LW) higher-order shear deformable beam theory (HSDBT) is presented for a thermal buckling analysis of simply-supported, functionally graded material beams under a uniform temperature change. A bi-objective optimization of functionally graded beams in a thermal environment is presented to maximize the critical temperature changes and to minimize their total mass using a non-dominated sorting-based genetic algorithm (GA).

1. INTRODUCTION

Functionally graded material beams are emerging heterogeneous material structures, which are formed by mixing two-phase materials (i.e., ceramic and metal materials) with a pre-designed spatial distribution of volume fractions of the constituents. The material properties of functionally graded structures gradually and smoothly vary through their domain, so that some drawbacks can be prevented, such as delamination and stress concentration, which usually occur at the interfaces between the adjacent layers for laminated composite structures due to the material properties suddenly changing at these locations. Functionally graded beams provide a large optimal design space in engineering practice. Engineers can design the spatial distributions of the volume fractions of the constituents according to practice demands to obtain the best structural physical properties, such that functionally graded beams are also becoming increasingly more popular in various high-end industries (Koizumi, 1997). A variety of mechanical analyses of functionally graded material structures and multi-objective optimization of these structures are thus attracting considerable attention.

In this work, the authors aim at investigating the material composition optimization of a functionally graded beam under a uniform temperature change in order to maximize the critical temperature change and to minimize the total mass of the functionally graded beam. A non-dominated sorting-based GA (Deb, 2002) is used for the current bi-objective

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optimization analysis in the current issue. The through-thickness distribution of the material properties of the functionally graded beam is assumed to be a three-parameter power-law function of the volume fractions of the constituents (Tornabene and Viola, 2007), the material-property gradient indices of which are thus to be determined for the optimal material profile.

2. *THE NON-DOMINATED SORTING-BASED GA*

In this work, the authors consider the bi-objective optimization of the volume fractions of the constituents of functionally graded material beams under a uniform temperature change in order to maximize the critical temperature change of the FG beam and to minimize its total mass. The thickness and the length of the beam are denoted as *h* and *L*, respectively. In the analysis, the beam is artificially divided into N_l layers, and the thickness of each individual layer constituting the beam is h_m (m=1-N_l), such that $\sum_{m=1}^{N_l} h_m = h$.

The functionally graded beam is considered to be formed by mixing a metal material and a ceramic material according to a three-parameter power-law function of the volume fractions of the constituents through the thickness coordinate of the functionally graded beam, which is given as follows:
The three-parameter power-law function,

$$\Gamma_c = \left[\left(\frac{1}{2} + \frac{z}{h} \right) + \kappa_b \left(\frac{1}{2} - \frac{z}{h} \right)^{\kappa_c} \right]^{\kappa_p} \quad \text{and} \quad \Gamma_c + \Gamma_m = 1, \tag{1}$$

where the symbols κ_b, κ_c, and κ_p denote the material-property gradient indices. The material at the top surface of the functionally graded beam (i.e., $z=h/2$) is ceramic rich, and when $\kappa_b = 0$, the material at the bottom surface of the functionally graded beam (i.e., $z=-h/2$) is metal rich.

In the optimal design, a mass ratio R_m is defined as follows:

$$R_m = \left(\hat{\rho}_f - \hat{\rho}_c \right) / \left(\hat{\rho}_m - \hat{\rho}_c \right), \tag{2}$$

where $\hat{\rho}_f$, $\hat{\rho}_c$, and $\hat{\rho}_m$ are the total mass per unit area in *x*-*y* plane of the functionally graded beam, the homogeneous ceramic material, and the homogeneous metal material, respectively. $\hat{\rho}_k = \int_{-h/2}^{h/2} \rho_k(z) dz$, in which $k=f$, c, or m.

The critical temperature parameter $\Delta \hat{T}_{cr}$ is defined as follows:

$$\Delta \hat{T}_{cr} = \Delta T_{cr} \, \alpha_c \left(L / h \right)^2, \tag{3}$$

where α_c denotes the thermal expansion coefficient of a reference ceramic material. The relevant thermal buckling analysis of the functionally graded beams is obtained using an LW HSDBT.

The objective functions are defined as follows:

Objective function 1: $F_1 = R_m$, (4)

Objective function 2: $F_2 = 1 - \left\{ \left[(\Delta T_{cr}) - (\Delta T_{cr})_c \right] / \left[(\Delta T_{cr})_m - (\Delta T_{cr})_c \right] \right\}$, (5)

where the ranges of F_1 and F_2 are $0 \le F_1 \left(\text{or } F_2 \right) \le 1$.

Because non-dominated sorting is used in the current GA, the values of these objective functions given in eqs. (4) and (5) are thus used to classify each design into its corresponding non-dominated front, which is also the assigned fitness value used for the sorting process. Minimization of the fitness function can be accomplished when the critical temperature change parameters of the functionally graded beam are obtained.

The non-dominated sorting-based GA is used as an optimal technique, the usual form of which is described as follows: The GA starts with an initial set of random solutions, which are called a population. Each individual in the population is called a chromosome, which represents a solution to the problem at hand. The chromosomes are made of discrete units called genes. Each gene controls one or more features of the chromosome. The chromosomes evolve through successive iterations, called generations. Based on some measures of fitness, the chromosomes during each generation are evaluated. To create the next generation, new chromosomes in the next generation, called offspring, are formed using two operators, which are the crossover and mutation operators. In the former, some portions of two chromosomes in the current generation are merged together, and in the latter, some portions of chromosomes in the current generation are modified. Thus, the crossover operator leads the population to converge by means of making the chromosome in the population alike, and the mutation operator assists the search escaping from local optima by reintroducing genetic diversity back into the population. A new generation is formed by selecting and rejecting some of the parents and offspring according to their fitness values so as to keep the population size of each generation constant. After several generations, the GA converges to the best chromosome, representing the optimum solution to the problem considered.

3. NUMERICAL EXAMPLES

A bi-objective optimization for material composition of a simply-supported functionally graded beam subjected to a uniform temperature change is considered in order to maximize its critical temperature change and minimize its self-weight. In the optimal design, the functionally graded beam is considered to be a two-phase composite material, where one phase is the ceramic material (ZrO_2) and the other is the metal material (SUS304). The temperature-dependent material properties of ZrO_2 and SUS304 are given in Table 1 (Shen, 2009). The material properties of ZrO_2 and SUS304 with the temperature variable vary from 300K to 1100K. The material properties of the functionally graded beam are assumed to obey a three-parameter power-law distribution of volume fractions of the constituents along the thickness of the functionally graded beam, and the effective material properties are estimated using the rule of mixtures. The non-dominated sorting-based GA is used to determine some sets of Pareto-optimal solutions of the undetermined coefficients κ_p, κ_b, and κ_c, in which 200 initial populations are randomly generated, in which the ranges of κ_p, κ_b, and κ_c are taken as $0 \le \kappa_p \le 50$, $0 \le \kappa_b \le 1$, and $1 \le \kappa_c \le 3$, respectively.

Table 1. Temperature dependent material properties $P(T)$ of the metal material (SUS304) and the ceramic material (ZrO_2), where $P(T)=P_0\,(P_{-1}\,T^{-1} + 1 + P_1\,T + P_2\,T^2 + P_3\,T^3)$ (Shen, 2009).

Materials	P_0	P_{-1}	P_1	P_2	P_3	P at 300K
ZrO_2						
E	244.27e+9	0	-1.371e-3	1.214e-6	-3.681e-10	168.06e+9
α	12.766e-6	0	-1.491e-3	1.006e-5	-6.778e-11	18.591e-6
υ	0.2882	0	1.133e-4	0	0	0.298
ρ	3657	0	0	0	0	3657
SUS304						
E	201.04e+9	0	3.079e-4	-6.534e-7	0	207.79e+9
α	12.330e-6	0	8.086e-4	0	0	15.321e-6
υ	0.3262	0	-2.002e-4	3.797e-7	0	0.318
ρ	8166	0	0	0	0	8166

Figure 1 shows the populations at the initial and 20th generations, where the temperature-dependent material properties are considered, respectively. It can be seen in Fig. 2 that the non-dominated, sorting-based GA converges rapidly and that the Poreto-optimal solutions can be yielded after 20 generations. As a result, the Pareto-optimal solutions can be sorted by the F_2 function values from the largest to the smallest. These Pareto-optimal solutions may provide design engineers with valuable information regarding what set of the material-property gradient indices (κ_p, κ_b, and κ_c) they need according to the weight number ratio of (w_2/w_1).

4. CONCLUSIONS

In this work, the authors developed a mixed LW HSDBT for the thermal buckling analysis of FG beams subjected to a uniform temperature change, and then they further developed a non-dominated sorting-based GA for bi-objectives optimization of the material composition of a three-parameter FG beam, in which the temperature-dependent material properties are considered.

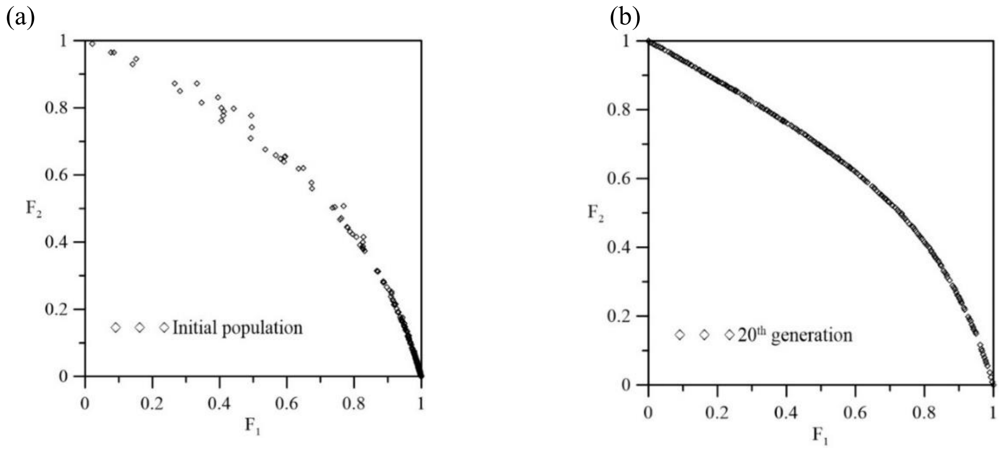

Fig. 1. 200 populations for different generations shown in the objective spaces, in which the TD material properties are considered; (a) initial generation; (b) 20th generation (Wu and Li, 2021).

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