Bi-objective Optimization of Functionally Graded Beams in a Thermal Environment

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ABSTRACT

A mixed layer-wise (LW) higher-order shear deformation theory (HSDT) is developed for the thermal buckling analysis of simply-supported, functionally graded (FG) beams subjected to a uniform temperature change. A bi-objective optimization of FG beams in a thermal environment is presented to maximize the critical temperature change parameters and to minimize their total mass using a non-dominated sorting-based genetic algorithm (GA).

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INTRODUCTION

Functionally graded (FG) beams are emerging heterogeneous material structures, which are formed by mixing two- or multiple-phase materials with a pre-designed spatial distribution of volume fractions of the constituents. Because the material properties of FG structures gradually and smoothly vary through their domain, some drawbacks can be prevented, including delamination and stress concentration, which usually occur at the interfaces between adjacent layers in the case of the laminated composite structures due to the material properties suddenly changing at these locations.

Some comprehensive literature surveys with regard to articles examining the mechanical analyses of FG structures can be found in the public literature (Jha et al., 2013; Wu et al., 2008). Some articles related to the optimal design of FG beams have been presented using different single-and multi-objective optimization algorithms combined with the advanced and refined shear deformation beam theories (Goupee and Vel, 2006; Tornabene and Ceruti, 2013).

In this work, the authors aim at investigating the material composition optimization of an FG beam under a uniform temperature change in order to maximize the critical temperature change parameter and minimize the total mass of the FG beam. A non-dominated sorting-based GA (Deb, 2002) is used for the current bi-objective optimization analysis in the current issue. The through-thickness distribution of the material properties of the FG beam is assumed to be a three-parameter power-law function of the volume fractions of the constituents (Tornabene and Viola, 2007), the material-property gradient indices of which are thus to be determined for the optimal material profile.

THE NON-DOMINATED SORTING-BASED GA

In this work, the authors consider the bi-objective optimization of the volume fractions of the constituents of FG beams subjected to a uniform temperature change in order to maximize the critical temperature change parameter of the FG beam and to minimize its total mass. The configuration and coordinates of the FG beam are shown in Fig. 1, in which h and L represent the thickness and the length of the FG beam, respectively. In the analysis, the FG beam is artificially divided into N_l layers, and the thickness of each individual layer constituting the beam is h_m (m=1-N_l), such that $\sum_{m=1}^{N_l} h_m = h$.

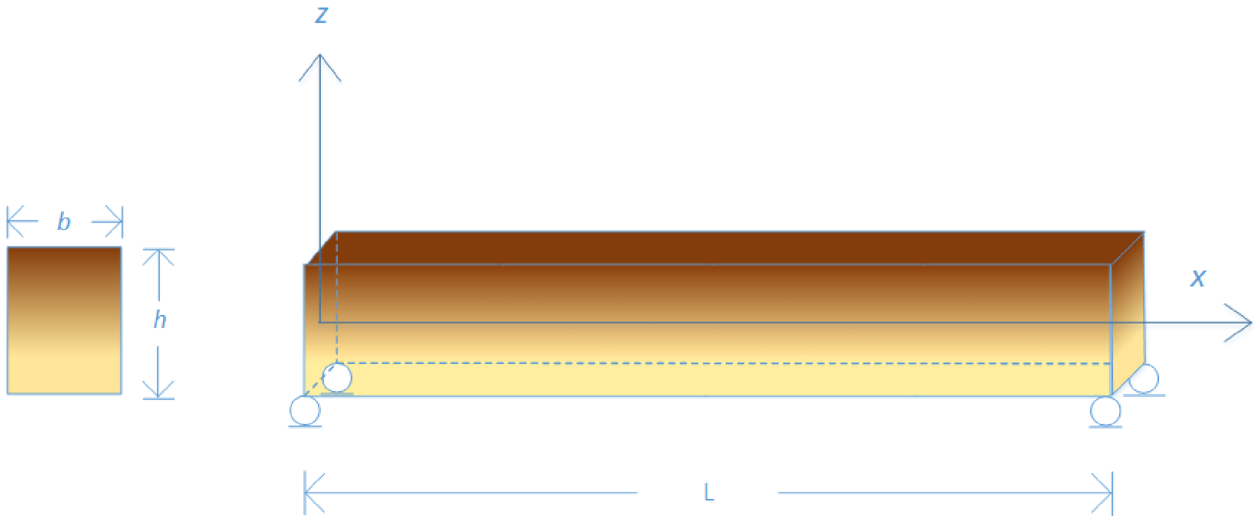

Fig. 1 Configuration and coordinates of a simply-supported FG beam.

The FG beam is considered to be formed by mixing a metal material and a ceramic material according to a three-parameter power-law function of the volume fractions of the constituents through the thickness coordinate of the FG beam, which is given as follows:

The three-parameter power-law function,

$$\Gamma_c = \left[\left(\frac{1}{2} + \frac{z}{h} \right) + \kappa_b \left(\frac{1}{2} - \frac{z}{h} \right)^{\kappa_c} \right]^{\kappa_p} \quad \text{and} \quad \Gamma_c + \Gamma_m = 1, \tag{1}$$

where the symbols κ_b, κ_c, and κ_p denote the material-property gradient indices. The material at the top surface of the FG beam (i.e., $z=h/2$) is ceramic rich, and when $\kappa_b = 0$, the material at the bottom surface of the FG beam (i.e., $z=-h/2$) is metal rich.

In the optimal design, a mass ratio R_m is defined as follows:

$$R_m = \left(\hat{\rho}_f - \hat{\rho}_c \right) / \left(\hat{\rho}_m - \hat{\rho}_c \right), \tag{2}$$

where $\hat{\rho}_f$, $\hat{\rho}_c$, and $\hat{\rho}_m$ are the total mass per unit area in *x*-*y* plane of the FG beam, the homogeneous ceramic material, and the homogeneous metal material, respectively. $\hat{\rho}_k = \int_{-h/2}^{h/2} \rho_k(z)\, dz$, in which *k*=*f*, *c*, or *m*.

The objective functions are defined as follows:

Objective function 1: $F_1 = R_m$, (3)

Objective function 2: $F_2 = 1 - \left\{ \left[\left(\Delta T_{cr} \right) - \left(\Delta T_{cr} \right)_c \right] / \left[\left(\Delta T_{cr} \right)_m - \left(\Delta T_{cr} \right)_c \right] \right\}$, (4)

where the ranges of F_1 and F_2 are $0 \le F_1 \left(\text{or } F_2 \right) \le 1$.

NUMERICAL EXAMPLES

A bi-objective optimization for material composition of a simply-supported FG beam subjected to a uniform temperature change is considered in order to maximize its critical temperature change and minimize its self-weight. In the optimal design, the FG beam is considered to be a two-phase composite material, where one phase is the ceramic material (ZrO_2) and the other is the metal material (SUS304). The temperature-dependent material properties of ZrO_2 and SUS304 are given in Table 1 (Shen, 2009). The material properties of ZrO_2 and SUS304 with the temperature variable vary from 300K to 1100K. The material properties of the FG beam are assumed to obey a three-parameter power-law distribution of volume fractions of the constituents along the thickness of the FG beam, and the effective material properties are estimated using the rule of mixtures. The non-dominated sorting-based GA is used to determine some sets of Pareto-optimal solutions of the undetermined coefficients κ_p, κ_b, and κ_c, in which 200 initial populations are randomly generated, in which the ranges of κ_p, κ_b, and κ_c are taken as $0 \le \kappa_p \le 50$, $0 \le \kappa_b \le 1$, and $1 \le \kappa_c \le 3$, respectively.

Table 1. Temperature dependent material properties $P(T)$ of the metal material (SUS304) and the ceramic material (ZrO_2), where $P(T)=P_0\,(P_{-1}\,T^{-1} + 1 + P_1\,T + P_2\,T^2 + P_3\,T^3)$.

Materials	P_0	P_{-1}	P_1	P_2	P_3	P at 300K
ZrO_2						
E	244.27e+9	0	-1.371e-3	1.214e-6	-3.681e-10	168.06e+9
α	12.766e-6	0	-1.491e-3	1.006e-5	-6.778e-11	18.591e-6
υ	0.2882	0	1.133e-4	0	0	0.298
ρ	3657	0	0	0	0	3657
SUS304						
E	201.04e+9	0	3.079e-4	-6.534e-7	0	207.79e+9
α	12.330e-6	0	8.086e-4	0	0	15.321e-6
υ	0.3262	0	-2.002e-4	3.797e-7	0	0.318
ρ	8166	0	0	0	0	8166

Figure 2 shows the populations at the initial, first, second, fifth, tenth, and 20th generations, where the temperature-dependent material properties are considered, respectively. It can be seen in Fig. 2 that the non-dominated, sorting-based GA converges rapidly and that the Poreto-optimal solutions can be yielded after 20 generations. As a result, the Pareto-optimal solutions can be sorted by the F_2 function values from the largest to the smallest. These Pareto-optimal solutions may provide design engineers with valuable information regarding what set of the material-property gradient indices (κ_p, κ_b, and κ_c) they need according to the weight number ratio of (w_2/w_1).

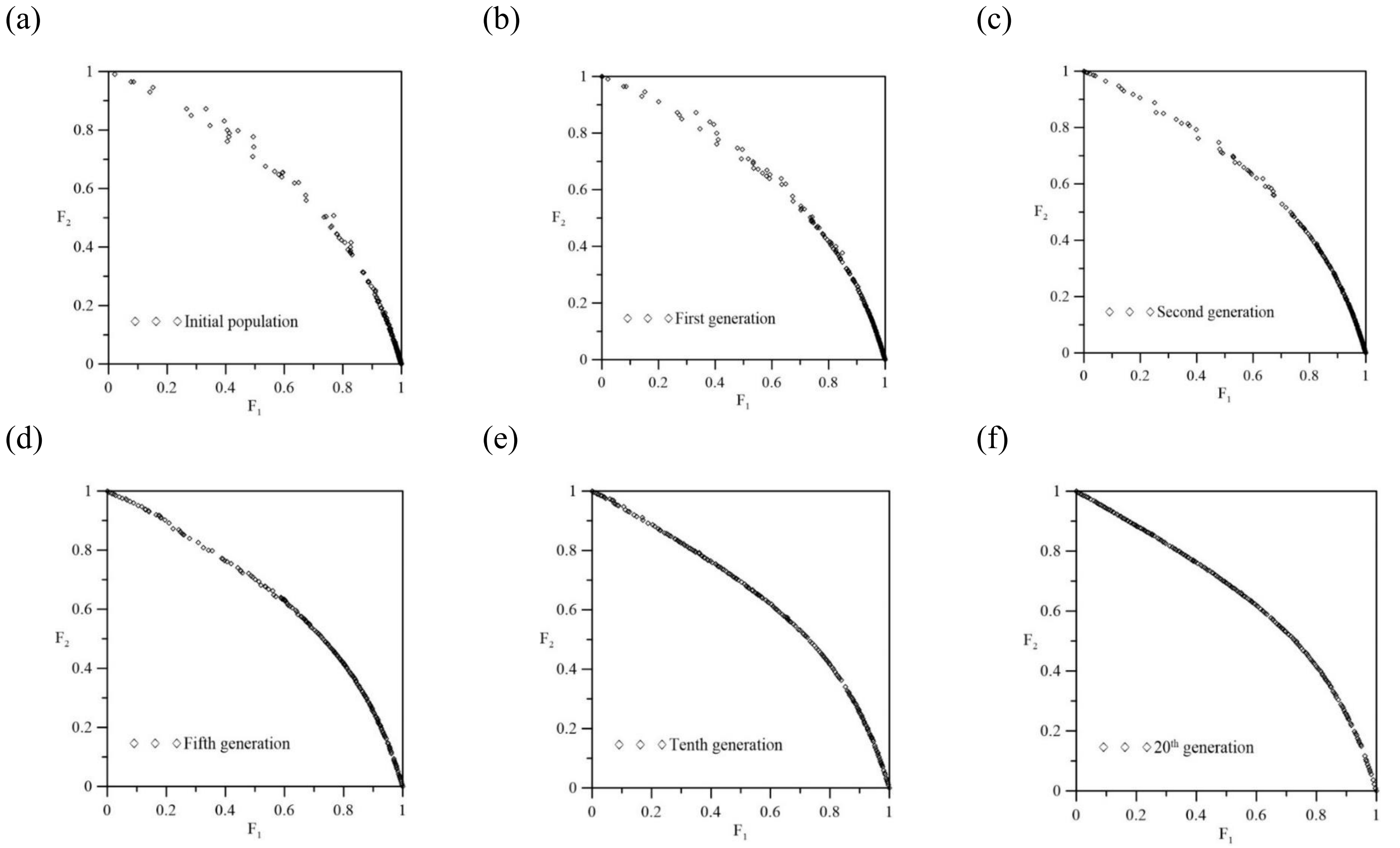

Fig. 2. 200 populations for different generations shown in the objective spaces, in which the TD material properties are considered; (a) initial generation; (b) 1st generation; (c) 2nd generation; (d) 5th generation; (e) 10th generation; (f) 20th generation.

CONCLUSIONS

In this work, the authors developed a mixed LW HSDT for the thermal buckling analysis of FG beams subjected to a uniform temperature change, and then they further developed a non-dominated sorting-based GA for bi-objectives optimization of the material composition of a three-parameter FG beam, in which the temperature-dependent material properties are considered.

In the numerical examples, the optimal sets of the material-property gradient indices of a three-parameter FG beam are obtained to maximize the critical temperature change parameter it can withstand and to minimize the FG beam's self-weight. The results show the self-developed non-dominated sorting-based GA converges rapidly, where the Poreto-optimal solutions can be yielded after the 20th generation. The non-dominated sorting-based GA can also be extended to other optimal designs of FG beams with multiple objective functions.

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