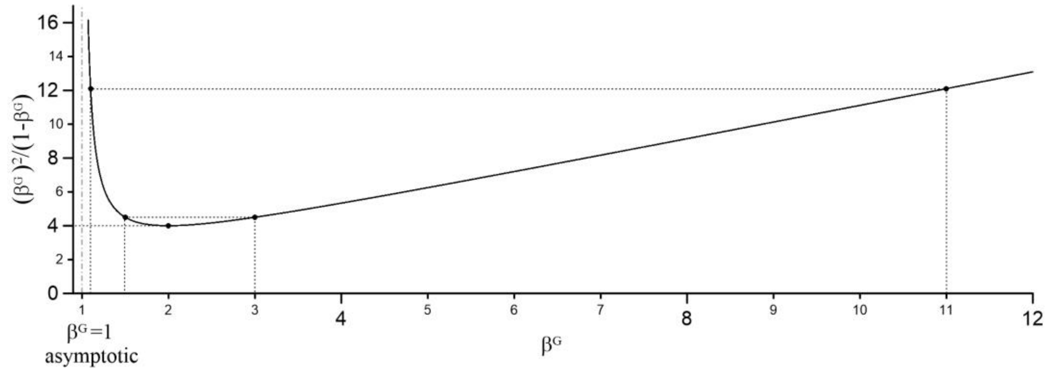


Fig. 2 $(\beta^G)^2 / \beta^G - 1$ coefficient

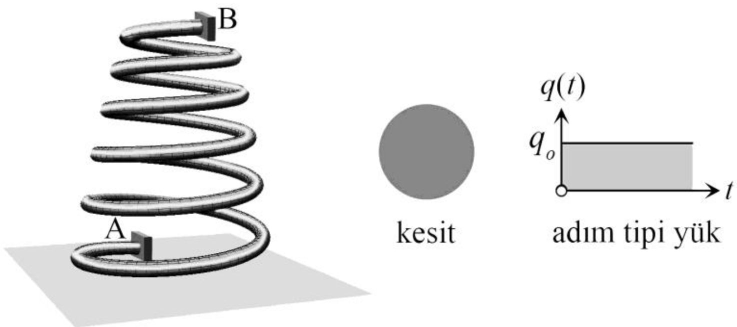
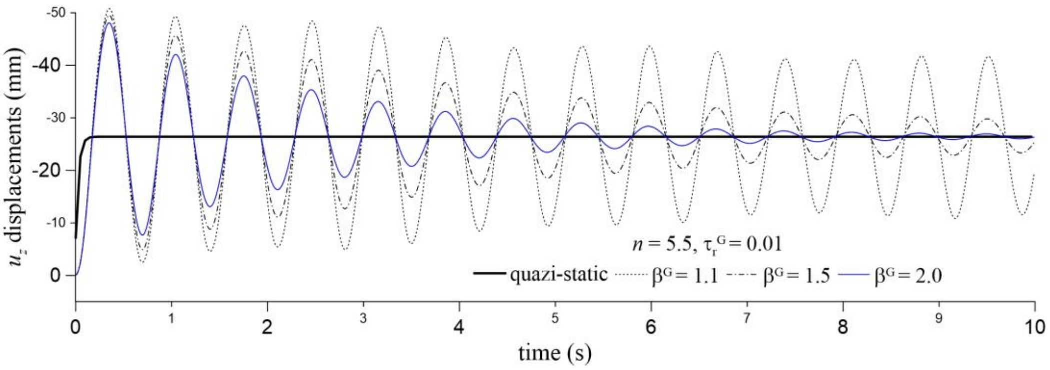


Fig. 3 The conical helicoidal rod having circular cross-section.

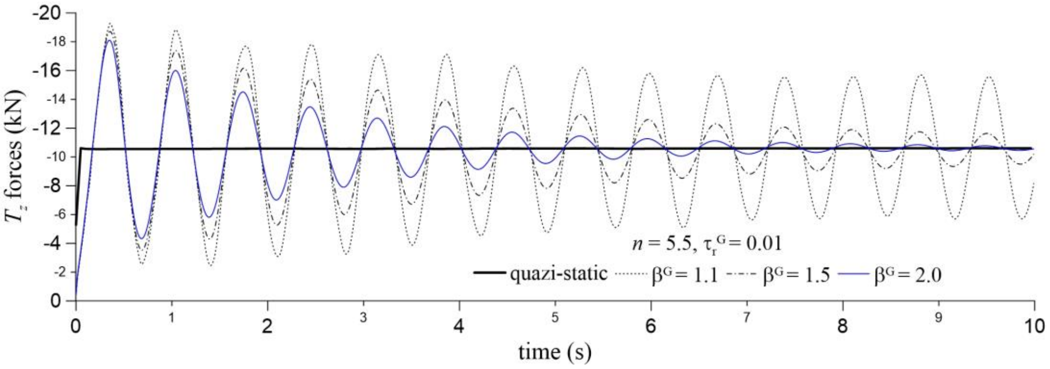
3. NUMERICAL EXAMPLES

A viscoelastic conical helicoidal rod with fixed-fixed boundary condition is solved (see Fig. 3). The helix geometry has $n = 5.5$ number of active turns, the height of the rod is $H = 5\,\text{m}$ and the minimum radius of helix to maximum of helix ratios $R_{\min} / R_{\max} = 0.5$ where $R_{\max} = 2\,\text{m}$. The circular cross-sectional diameter of the rod is $D = 20\,\text{cm}$. The viscoelastic material exhibits the standard type of distortional behavior while having elastic Poisson's ratio $\bar{\upsilon} = \upsilon = 0.3$. The material parameters are $G = 80\,\text{GPa}$, $\tau_r^G = 0.01\text{s}$, $\beta^G = 1.1, 1.5, 2.0, 3.0, 11.0$ and the density of material $\rho = 7850\ \text{kg/m}^3$. The form of the complex shear modulus can be obtained by using Eq. (3). The rod is subjected to a dynamic rectangular impulsive type of uniformly distributed load $q = q_z(t)$, where $q_o = 500\,\text{N/m}$ (see Fig. 3). The quasi-static and dynamic responses of the rod are determined within $0 \le t \le 10\,\text{s}$. The analyses are carried out in the Laplace space and the results are transformed back to the time space numerically by modified Durbin's algorithms (Dubner and Abate 1968, Durbin 1974, Narayanan 1980). The parameters which are used in the analysis for inverse Laplace transformation algorithm are $N = 2^{11}$ and $aT = 6$ which are verified by Eratlı et al. (2014).

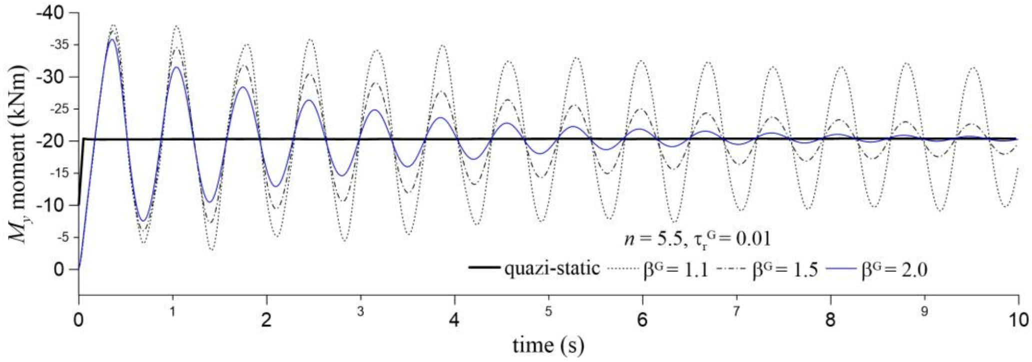
The vertical displacements u_z at the middle point of the helicoidal rod, the force T_z the moments M_y at the clamped end A (see Fig. 1) are determined using 100 finite elements through the analysis. The time variation curves of u_z , T_z , and M_y are depicted for $\beta^G = 1.1, 1.5, 2.0$ and $\beta^G = 2.0, 3.0, 11.0$ ratios in Figs. 4-5, respectively. The aim of these figures is to investigate the influence of viscoelastic material parameter β^G on the dynamic response of viscoelastic conical helicoidal rod. For the same values of τ_r^G retardation time, viscosity parameter η^G , and shear modulus G , there exist two different β^G ratios, except $\beta^G = 2$ that is a transition value ($G_1 = G_2$). One of these β^G ratios is between $1 < \beta^G < 2$ ($G_1 < G_2$) and the another one is $\beta^G > 2$ ($G_1 > G_2$) (see Fig. 2). G_1, G_2 values are related to viscosity part and elastic part of the standard model type viscoelastic material. In Figs. 4-5, the dashed, the dot and the solid lines are used for $\beta^G = 1.5, 3.0$, $\beta^G = 1.1, 11.0$ and $\beta^G = 2$ ratios, respectively. The responses curves of $\beta^G = 2$ values is lowest contour $\beta^G = 1.1, 1.5$ (see Fig. 4), whereas is highest contour $\beta^G = 3.0, 11.0$ (see Fig. 5). The amplitudes of u_z, T_z, and M_y (see Figs.4-5) decrease with the increasing β^G ratios, and approach to the quasi static case. When the amplitude of curves are compared Fig. 5 with respect to Fig. 4, it is observed that the amplitudes for $\beta^G > 2$ values dissipate more rapidly.



(a) u_z

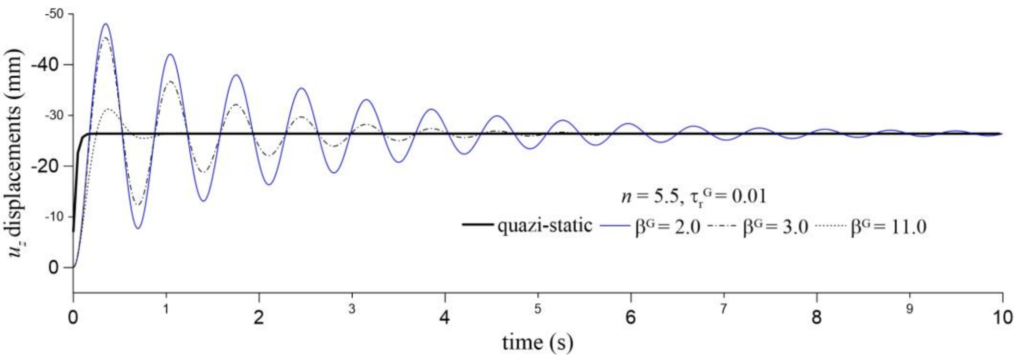


(b) T_z

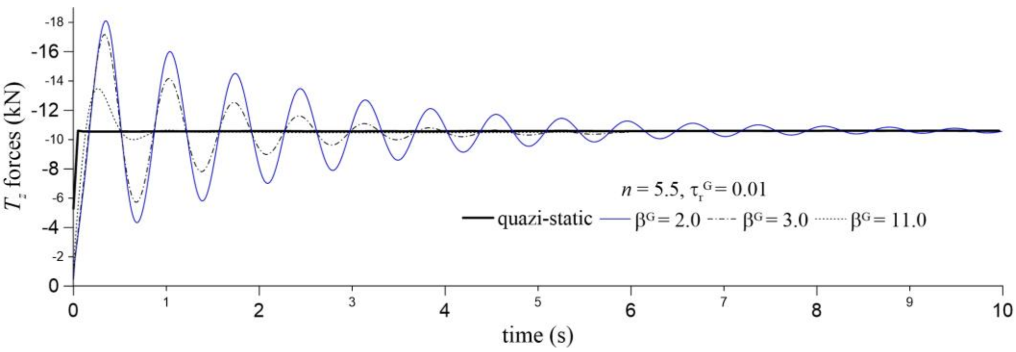


(c) M_y

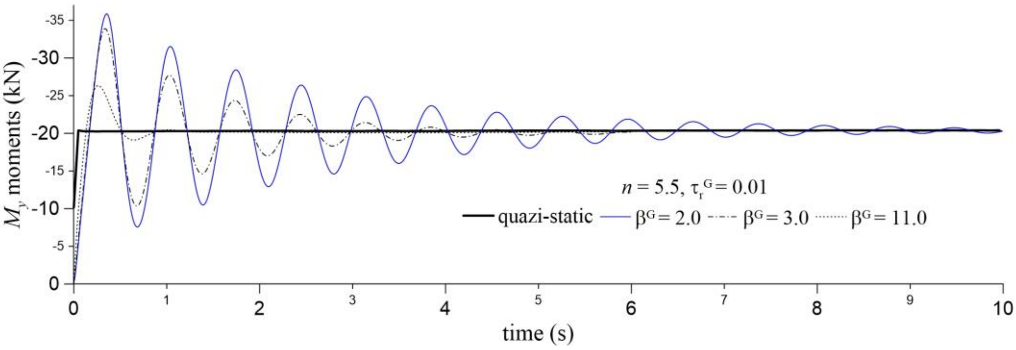
Fig. 4 Time histories of viscoelastic conical helicoidal rod for $\beta^G = 1.1, 1.5, 2.0$



(a) u_z



(b) T_z



(c) M_y

Fig. 5 Time histories of viscoelastic conical helicoidal rod for $\beta^G = 2.0, 3.0, 11.0$

4. CONCLUSION

The dynamic viscoelastic behavior under the step type of uniformly distributed dynamic load of conical helicoidal rod is examined for different viscosity parameter β^G ratio via the mixed finite element method. For this purpose, the viscoelastic material behavior is simulated by using the standard model while having elastic Poisson's ratio and the viscoelastic properties are accounted using the correspondence principle. The finite element solutions are carried out in the Laplace space. The results obtained in the Laplace space are transformed back to time space via modified Durbin's algorithm. The effects of the viscoelastic material parameter β^G ratios on the dynamic behavior of the conical helix are discussed extensively.

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