

number should be considered in preconditioning process for convergence acceleration of unsteady low Mach number flows, and showed that optimal scaling is required for spatial accuracy.

As a previous work, we developed two-phase shock-stable numerical schemes [5] that originate from the RoeM [6] and AUSMPW+ [7] schemes. We showed that these extended numerical flux schemes are robust and efficient for compressible two-phase flow and that they can deal with compressible-incompressible two-phase flow through the application of steady preconditioning techniques. They are not able to compute unsteady low Mach number flow accurately because they do not take into account unsteady preconditioning and scaling of numerical flux schemes.

The purpose of this paper is to compute unsteady cavitating flow around a 2-D wedge by using compressible two-phase URANS solver. For the computation of unsteady low Mach number flow, we extend the previous all-speed two-phase RoeM and AUSMPW+ numerical flux schemes to the application of unsteady system preconditioning. We also scale numerical dissipations of the numerical flux schemes separately from the system preconditioning.

2. NUMERICAL METHODS

2.1 Governing equation

The homogeneous mixture equation with mass fraction is adopted as the governing equation for unsteady cavitating flow. In homogeneous flow theory, the relative motion between phases is not considered; instead, mixture is treated as a pseudo-fluid whose properties are suitable averages of each component in the flow. This approach is based on the view that it is sufficient to describe each phase as a continuum obtained from a microscopic description using a suitable averaging process. In this model, continuity, momentum, and energy equations are used to describe the fluid mixture, while a single continuity equation is used for vapor and non-condensable gas phases. The governing equation is as follows,

$$\frac{\partial}{\partial t} \int_{\Omega} W d\Omega + \oint_{d\Omega} [F - F_v] dS = 0 \quad (1)$$

where W means the conservative variables vector. F and F_v stand for convective flux vector and viscous flux vector, respectively.

$$W = [\rho \quad \rho u \quad \rho v \quad \rho E \quad \rho y_v \quad \rho y_g]^T \quad (2)$$

$$F = [\rho U \quad \rho u U + n_x p \quad \rho v U + n_y p \quad \rho U H \quad \rho y_v U \quad \rho y_g U]^T \quad (3)$$

U is a contravariant velocity that is normal to the surface element dS . y_v and y_g stand for the mass fractions of vapor and non-condensable gas phase, respectively.

2.2 System preconditioning

At low speed flow, system stiffness resulting from disparate convective and acoustic velocities causes deterioration of convergence rates. Convergence rates can be made independent of the Mach number by altering the acoustic speed of the system such that all eigenvalues are of the same order and thus the condition number approaches unity. We precondition the governing equations (Eq. (1)) by pre-multiplying the time derivative term by the preconditioning matrix from Weiss and Smith [7] as follows

$$\Gamma \frac{\partial}{\partial \tau} \int_{\Omega} Q \, d\Omega + \oint_{d\Omega} [F - F_v] \, dS = 0 \quad (4)$$

Where Q stands for the primitive variable vector given by

$$Q = [p \quad u \quad v \quad T \quad y_v \quad y_g]^T \quad (5)$$

and the preconditioning matrix Γ is

$$\Gamma = \begin{bmatrix} \frac{1}{\beta} & 0 & 0 & \frac{\partial \rho}{\partial T} & \frac{\partial \rho}{\partial y_v} & \frac{\partial \rho}{\partial y_g} \\ \frac{u}{\beta} & \rho & 0 & \frac{\partial \rho}{\partial T} u & \frac{\partial \rho}{\partial y_v} u & \frac{\partial \rho}{\partial y_g} u \\ \frac{v}{\beta} & 0 & \rho & \frac{\partial \rho}{\partial T} v & \frac{\partial \rho}{\partial y_v} v & \frac{\partial \rho}{\partial y_g} v \\ H^* & \rho u & \rho v & \frac{\partial \rho}{\partial T} H + \rho \frac{\partial h}{\partial T} & \frac{\partial \rho}{\partial y_v} H + \rho \frac{\partial h}{\partial y_v} & \frac{\partial \rho}{\partial y_g} H + \rho \frac{\partial h}{\partial y_g} \\ \frac{y_v}{\beta} & 0 & 0 & \frac{\partial \rho}{\partial T} y_v & \frac{\partial \rho}{\partial y_v} y_v + \rho & \frac{\partial \rho}{\partial y_g} y_v \\ \frac{y_g}{\beta} & 0 & 0 & \frac{\partial \rho}{\partial T} y_g & \frac{\partial \rho}{\partial y_v} y_g & \frac{\partial \rho}{\partial y_g} y_g + \rho \end{bmatrix} \quad (6)$$

where

$$H^* = \frac{H}{\beta} + \rho \frac{\partial h}{\partial p} - 1 \quad (7)$$

The eigenvalues of the preconditioned system in equation (6) are given by

$$\lambda \left(\Gamma^{-1} \frac{\partial F}{\partial Q} \right) = U, U, U, U, U' - D, U' + D \quad (8)$$

where

$$U' = \frac{1}{2} \left(1 + \frac{c'^2}{c^2} \right) U \quad (9)$$

