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A general station-keeping strategy for deep space missions utilizing libration point orbits

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ABSTRACT

This paper is devoted to the study of applying numerical techniques to robustly control the collinear libration point orbits (LPOs). A new methodology is proposed exploiting the hyperbolic dynamics of the collinear libration points. Numerical tools are developed to facilitate the efficient LPO station-keeping process, which are also applicable to realistic force models and inherently parallelizable. After the precise formulation of the libration point region, the transformation from synodic coordinates to center manifold coordinates is derived; based on this transformation, the error state can be projected to center manifold by introducing maneuver Delta-V. Then, a manifold projection method is designed to optimally obtain the Delta-V. By applying the algorithm to the collinear libration points of Earth–Moon system, it has been demonstrated that the proposed method can handle the station-keeping problems of various types of LPOs in a unified manner and is robust over a wide range of energy levels.

1. INTRODUCTION

The variety of libration point orbits (LPOs) in the circular restricted three-body problem (CR3BP) forms a fundamental framework for preliminary trajectory design and offers a broad range of mission possibilities. Due to the hyperbolic component of the dynamics, all LPOs of the collinear libration points are inherently unstable. Uncontrolled flight in these libration point regions quickly leads to exponential divergence. As a consequence, station-keeping strategy is essential for any realistic periodic and quasi-periodic LPO mission.

Howell and Pernicka proposed the target point approach. While this approach doesn't rely on a specific orbit type, the cost function and the weighting matrix needed in the computation are in general difficult to obtain. In addition, the target point approach doesn't utilize the unique dynamical characteristics of the collinear libration points, which may lead to larger station-keeping fuel consumption.

Gómez et al. proposed the Floquet mode approach, which leverages the special

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dynamics of the collinear libration points, making this method more fuel-efficient. But the generation of the Floquet mode is based on the monodromy matrix computation, which in turn relies on the periodicity of the orbit. Thus the Floquet mode approach is not suitable for the station-keeping of quasi-periodic LPO missions.

In 1998, Jorba et al. adopted the normal form method to find the center manifold around libration points. This method is usually known as the reduction to the center manifold, it can be used to systematically find LPOs. Along with the computation of the center manifold, a coordinate transformation that can decouple the unstable dynamics is also calculated. We have found that by leveraging this coordinate transformation, a new kind of station-keeping strategy can be derived.

This paper presents a general LPO station-keeping strategy based on the reduction to the center manifold process. The structure of this paper is organized as follows: In Section 2 we introduce the definition of the libration point region, where the proposed station-keeping strategy can be used. Section 3 deals with the computation of the coordinate transformation in the reduction to the center manifold process. In Section 4, station-keeping strategy based on center manifold projection is discussed. Section 5 contains the numerical simulation utilizing developed strategy and relevant discussions.

2. FORMULATION OF THE LIBRATION POINT REGION

The libration point region defines a 3-dimensional area where the proposed station-keeping strategy can be applied. For our purpose, we consider that the libration point region should encompass all the LPOs in a certain energy level, which means it should be larger than the corresponding center manifold. Another restriction is that when a given trajectory leaves this region, the unstable hyperbolic behavior should be dominant.

For sufficiently low energy levels, the libration point region is relatively easy to define. For example, a set of two planar or spherical surfaces located on both sides of the planar Lyapunov orbits along with parts of the zero velocity surfaces can be used. As the energy level increases, the zero velocity surfaces shrink, and the planar Lyapunov orbits become further expanded and deformed. The aforementioned definition of boundary may become inappropriate for catching all the diverging trajectories. Moreover, it may even be impossible to find a planar or spherical surface that lies between the Lyapunov orbits and the second primary.

In order to properly define the libration point region for an extended energy range, we propose an approach based on the geometry of planar Lyapunov orbit family. First, we observe that orbits of the planar Lyapunov family do not intersect each other, and orbits of lower energy are fully contained in the higher energy orbits. Hence, we can choose a higher energy Lyapunov orbit as the boundary of the libration point region in the planar CR3BP. Second, for a range of energy levels not too large, numerical simulations based on the reduction to the center manifold procedure have shown that the center manifold is approximately rotationally symmetric, which is also suggested by the rotational symmetric structure of ZVS near the “neck” region. Therefore, we choose a planar Lyapunov orbit of energy level $C_0 - \Delta C$ and rotate it around x-axis to form the three dimensional libration point region of energy level C_0 in spatial CR3BP. The determination of the energy difference value ΔC is not rigorous. As C_0 gets smaller (i.e. higher energy level), the geometry of the center manifold may deviate from a rotational symmetric geometry. Then the ΔC value should be enlarged to accommodate this deviation.

On the computation aspects, a series of evenly spaced points $[x_n, y_n]$ are selected from the planar Lyapunov orbit of energy level $C_0 - \Delta C$, and then transformed to the polar coordinate form with origin located at the corresponding libration point.

$$\begin{bmatrix} \theta_n \\ r_n \end{bmatrix} = \begin{bmatrix} \tan^{-1}\left(x_n - x_L, y_n\right) \\ \sqrt{\left(x_n - x_L\right)^2 + y_n{}^2} \end{bmatrix} \tag{1}$$

where, x_L is the x coordinate of the collinear libration point in synodic CR3BP frame. Using θ_n as independent variable and r_n as function value, one can construct an interpolation function of the form

$$r_{\text{int}}(\theta) \tag{2}$$

Then, based on the rotational symmetry, we can write the boundary-crossing criterion function as follows

$$\Sigma(x,y,z) = \sqrt{(x-x_L)^2 + y^2 + z^2} - r_{\text{int}}\left(\cos^{-1}\left(\frac{x-x_L}{\sqrt{(x-x_L)^2 + y^2 + z^2}}\right)\right) \tag{3}$$

with the libration point region corresponding to

$$\Sigma(x,y,z) < 0 \tag{4}$$

The boundary-crossing event can be detected by the sign change of $\Sigma(x,y,z)$ and subsequently refined by a bisection method.

3. COORDINATE TRANSFORMATION IN CENTER MANIFOLD NORMAL FORM COMPUTATION

During the reduction to the center manifold process, a coordinate transformation that can decouple the unstable dynamics is also calculated. Under the decoupled center manifold coordinate system $(q_1, p_1, q_2, p_2, q_3, p_3)$, the p_1 、 q_1 coordinates correspond to stable and unstable direction respectively. As the stable component p_1 will automatically die out, the unstable component q_1 is the dominant divergent factor. If the value of q_1 can be systematically computed by the coordinate transformation, a scheme can be derived to cancel it out by introducing a station-keeping delta v, making the spacecraft stay in the libration point region for a longer period.

Gómez,Jorba have introduced the transformation from center manifold coordinates to the CRTBP synodic coordinates. What we need here is the inversed transformation. According to the computation process of the Hamilton of the center manifold, the linear part of the inversed transformation can be obtained with the matrix inversion, while the non-linear part can be obtained with the following property of the Lie series.

$$\hat{H} = H + \{H,G\} + \frac{1}{2}\{\{H,G\},G\} + \frac{1}{3!}\{\{\{H,G\},G\},G\} + \cdots \quad (5)$$

Where H 、 $\hat{H}$ is the Hamilton before and after the transformation, $\{\cdot,\cdot\}$ is the passion bracket,for any smooth function $F(q,p)$ 、 $G(q,p)$, we have

$$\{F,G\} = \sum_{i=1}^{3}\left(\frac{\partial F}{\partial q_i}\frac{\partial G}{\partial p_i} - \frac{\partial F}{\partial p_i}\frac{\partial G}{\partial q_i}\right) \quad (6)$$

Assuming the computing order of the center manifold is N ,the generating functions $G_3, G_4, \cdots G_N$ can be computed with the method of Gómez, Jorba (1991). The first step of calculating the inversed transformation is to compute transformation corresponding to $-G_N$. According to the theory of Lie series, this transformation can be obtain by viewing very components in $(q_1, p_1, q_2, p_2, q_3, p_3)$ as a Hamilton H, and then applying Eq. (5):

$$q_i^{(N)} = q_i + \{q_i, -G_N\} + \frac{1}{2!}\{\{q_i, -G_N\}, -G_N\} + \frac{1}{3!}\{\{\{q_i, -G_N\}, -G_N\}, -G_N\} + \cdots \quad (7)$$

$$p_i^{(N)} = p_i + \{p_i, -G_N\} + \frac{1}{2!}\{\{p_i, -G_N\}, -G_N\} + \frac{1}{3!}\{\{\{p_i, -G_N\}, -G_N\}, -G_N\} + \cdots \quad (8)$$

Where $i = 1,2,3$, q_i , p_i are coordinates before transformation, $q_i^{(N)}$, $p_i^{(N)}$ are coordinates after transformation. Similarly, treating $q_i^{(N)}$ 、 $p_i^{(N)}$ as the Hamilton H, and applying Eq. (5) to $-G_{N-1}$, we get composite transformation corresponding to $-G_N, -G_{N-1}$:

$$q_i^{(N-1)} = q_i^{(N)} + \{q_i^{(N)}, -G_{N-1}\} + \frac{1}{2!}\{\{q_i^{(N)}, -G_{N-1}\}, -G_{N-1}\} + \cdots \quad (9)$$

$$p_i^{(N-1)} = p_i^{(N)} + \{p_i^{(N)}, -G_{N-1}\} + \frac{1}{2!}\{\{p_i^{(N)}, -G_{N-1}\}, -G_{N-1}\} + \cdots \quad (10)$$

Following the same route, the final inversed transformation corresponding to generating functions $-G_N, -G_{N-1}, \cdots, -G_3$ can be obtained.

4. STATION-KEEPING BY PROJECTING STATES TO THE CENTER MANIFOLD

Denote a error state inside the libration point region as $X_e = (x_e, y_e, z_e, vx_e, vy_e, vz_e)$. According to the inversed transformation of section 3, the center manifold coordinates corresponding to X_e can be computed, which is denoted as $(q_1, p_1, q_2, p_2, q_3, p_3)$. The first component q_1 offers a means to measure the instability of X_e:

$$q_1 = q_1(X_e) \tag{11}$$

A station-keeping maneuver Δv is introduced at X_e by canceling the q_1 component out. Taking the design of 2-axis x-y controller as an example, the Δv is represented as $(\delta v_x, \delta v_y)$, and the q_1 component after the maneuver is

$$q_1 = q_1\left((x_e, y_e, z_e, vx_e + \delta v_x, vy_e + \delta v_y, vz_e)\right) \tag{12}$$

Setting q_1 to 0,we get a relationship that satisfied by $\delta v_x, \delta v_y$:

$$q_1\left((x_e, y_e, z_e, vx_e + \delta v_x, vy_e + \delta v_y, vz_e)\right) = 0 \tag{13}$$

Then the optimal Δv can be computed numerically by a general constrained optimization procedure.

5. NUMERICAL SIMULATION AND ANALYSIS

5.1 Station-keeping of Lissajous trajectory

In the following simulation, a Lissajous trajectory of earth-moon L_1 point is chosen as the reference trajectory. The initial state in CRTBP synodic coordinates is X_0(0.829, -0.000393, -0.0476, 0.00279, 0.1077, -0.04066). The maximum amplitudes in x, y, z direction are A_x=8253km , A_y=25140km , A_z=18335km respectively.

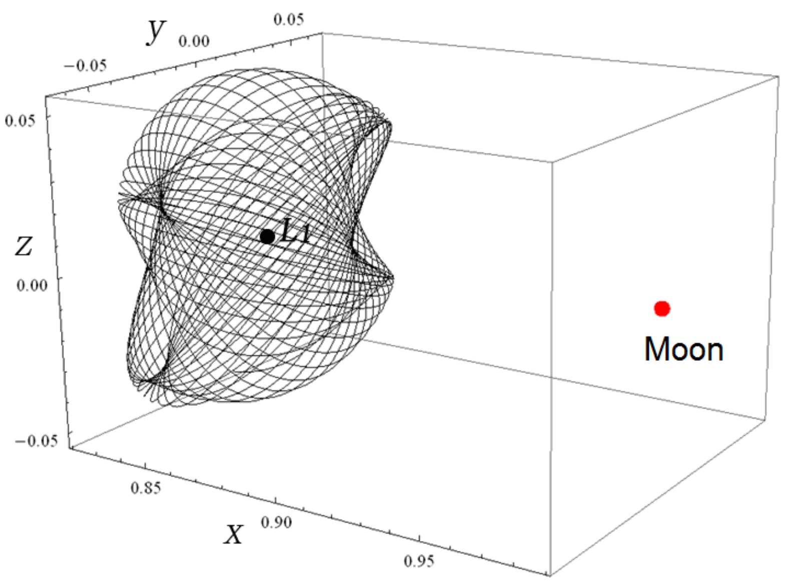

Fig.1 The reference Lissajous trajectory used in the simulation

Without station-keeping maneuver, the q_1 unstable component tends to increase exponentially, which can be clearly seen in Table 1.

Table. 1 Orbital states expressed in center manifold coordinates

t	q_1	p_1	q_2	p_2	q_3	p_3
0	0.0000337963	0.000052645	0.0000193309	0.456657	0.477664	0.178919
0.5	0.000976189	0.000103956	0.403984	0.239483	0.413708	0.283534
1	0.00350324	0.000166302	0.356949	0.23886	0.0750113	0.478553
1.5	0.0171132	0.000254882	0.132355	0.419008	0.44577	0.0802302
2	0.0730465	0.000158375	0.487655	0.00682569	0.233545	0.384257
2.5	0.27692	0.0000315699	0.236857	0.431862	0.25347	0.378311
3	0.997551	0.0161548	0.26608	0.404465	0.482882	0.00316409

Assuming a maneuver $(\delta v_x, \delta v_y)$ is introduced at $t=2$, the orbital states before the maneuver is：

$$X_{t=2} = (0.841653, -0.0497382, 0.0241285, -0.0498749, -0.00818304, -0.085897)$$

Using the inversed coordinate transformation (computed up to order 6), we get q_1

$$\begin{aligned} q_1(\delta v_x, \delta v_y) = {} & -74.795\delta v_x^5 \delta v_y + 114.138\delta v_x^4 \delta v_y^2 + 23.427\delta v_x^4 \delta v_y - \\ & 242.38\delta v_x^3 \delta v_y^3 - 5.19259\delta v_x^3 \delta v_y^2 - 2.946\delta v_x^3 \delta v_y - 331.452\delta v_x^2 \delta v_y^4 + \\ & 134.717\delta v_x^2 \delta v_y^3 - 20.906\delta v_x^2 \delta v_y^2 + 10.998\delta v_x^2 \delta v_y - 184.57\delta v_x \delta v_y^5 + \\ & 126.663\delta v_x \delta v_y^4 - 35.496\delta v_x \delta v_y^3 + 12.307\delta v_x \delta v_y^2 - 2.7184\delta v_x \delta v_y - \\ & 12.1934\delta v_x^6 + 19.197\delta v_x^5 - 3.9524\delta v_x^4 + 0.9902\delta v_x^3 - 4.5667\delta v_x^2 + \\ & 2.93986\delta v_x - 7.31865\delta v_y^6 + 22.0542\delta v_y^5 - 10.504\delta v_y^4 + 5.61\delta v_y^3 - \\ & 2.95318\delta v_y^2 + 1.73047\delta v_y - 0.0730465 \end{aligned} \tag{14}$$

Setting q_1 to 0, we get a implicit curve satisfied by δv_x and δv_y.

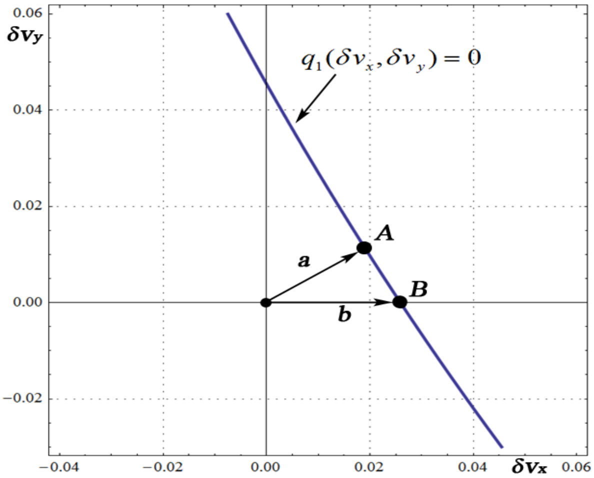

Fig .2 Implicit curve defined by $q_1(\delta v_x, \delta v_y) = 0$

In order to obtain the minimum $(\delta v_x, \delta v_y)$ value satisfying constraint $q_1(\delta v_x, \delta v_y) = 0$, a general numerical constrained optimization procedure is used. The result is:

$$\delta v_x = 19.5351 m/s, \delta v_y = 11.5619 m/s, |(\delta v_x, \delta v_y)| = 22.7 m/s$$

Assuming the time interval between consecutive station-keeping maneuver is fixed to 1 non-dimensional time unit (corresponding to 4.34913 days), all of the station-keeping maneuver can be computed accordingly. Setting the total flight time to $T = 10$, Table 2 sums up the Δv consumption for the 1 to 3 axes controller.

Table. 2 Station-keeping Δv (m/s) profiles for 3 kinds of controllers using the center manifold projection method

Maneuver sequence	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	$\sum \Delta v_i$
x axis controller	1.447	0.925	1.214	1.139	1.093	0.476	2.539	2.587	0.307	0.412	12.14
x-y axes controller	1.314	0.785	1.141	0.948	0.972	0.452	1.998	2.449	0.311	0.366	10.73
x-y-z controller	1.288	0.779	1.131	0.930	0.957	0.446	1.949	2.407	0.305	0.357	10.55

Figure 3 shows the q_1 curve of the x-y axes controller, indicating that the unstable component is indeed being cancelled out by the center manifold projection method.

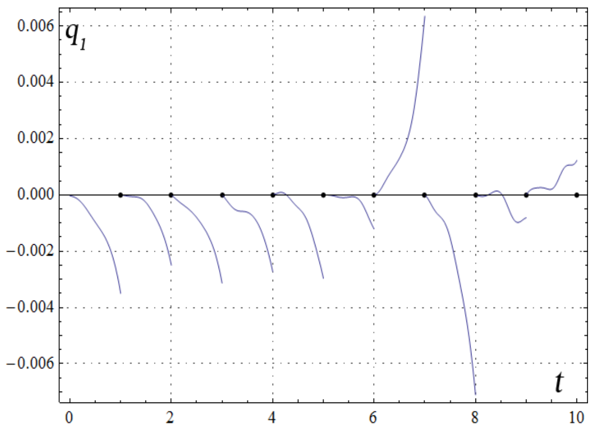

Fig. 3 q_1 curve of the simulation process using x-y axes controller

5.2 Comparison with Floquet mode approach

As the proposed station-keeping strategy doesn't rely on a specific orbit type to work, it's also suitable for the control of halo type orbits. In this case, we can compare its performance with the Floquet mode approach. The reference orbit is an earth-moon L_1 point halo orbit with x, y, z amplitudes $A_x = 10059km$ 、 $A_y = 26355km$ 、 $A_z = 16039km$ respectively.

Assuming the time interval between consecutive station-keeping maneuver is fixed to 7 days, the first 10 Δv s are calculated with different center manifold computation precision (higher computation order corresponding to higher precision). Table 3 sums up the Δv consumptions for these simulations.

It's evident that the Δv consumptions are closely related to the center manifold computation order. As the order increases, the center manifold computation becomes more precise, and the total Δv consumption decreases quickly. When the order of center manifold computation is larger than 13, the Δv consumption is significantly lower than the Floquet mode approach. In the case of order 15, the Δv consumption is about half of the Floquet mode approach.

Table. 3 Station-keeping Δv comparisons between Floquet mode approach and center manifold projection method

Maneuver sequence		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	$\sum \Delta v_i$
Floquet mode approach		1.752	0.175	0.109	0.206	0.325	0.117	0.168	0.069	0.213	0.265	3.403
center manifold projection method	Order 6	1.547	16.55	6.184	19.11	8.29	7.579	9.90	19.71	10.7	14.03	113.6
	Order 8	1.722	2.618	2.781	2.33	1.394	1.599	1.376	2.599	1.299	2.398	20.12
	Order 10	1.75	0.34	0.009	0.566	0.506	0.275	0.189	0.444	0.203	0.386	4.671
	Order 13	1.754	0.024	0.054	0.074	0.023	0.017	0.01	0.029	0.022	0.101	2.109
	Order 15	1.754	0.004	0.004	0.021	0.007	0.001	0.108	0.01	0.009	0.028	1.945

6. CONCLUSIONS

By taking advantage of the special dynamical structure of the collinear libration points, the definition of the libration point region is formulated; the transformation from synodic coordinates to the decoupled center manifold coordinates is derived. Based on this transformation, the error state can be projected to the nearest center manifold by introducing maneuver delta V. In this way, the unstable component can be canceled. The proposed station-keeping strategy fully integrates the dynamical property of collinear libration points, and is suitable for all kinds of libration point orbits. Finally, numerical simulations were performed on Lissajous orbit and halo orbit, verifying that this strategy can achieve good station-keeping performance.

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