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Damping of a Taut Cable attached with Near-support Damper and Mass

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ABSTRACT

This paper studies the damping and frequency of a taut cable with a damper and a concentrate mass. The complex characteristic equation of this cable-damper-mass system are obtained based on the taut string theory and by considering the compatibility requirements on each constraint point. By using a transfer matrix method. Asymptotic approximate solutions for damper and mass close to the cable end are developed provided that both the non-dimensional mass coefficient and the frequency shift between the free and damped cable system are small. The influences of the non-dimensional mass coefficient and its location on the maximum cable vibration damping, the optimal damper constant, and the corresponding frequency are also studied as both the damper and mass is installed near the cable anchorage. It is found that the mass will significantly increases the system damping when it was attached to nearby the damper when mass value are smaller than a critical value.

Keywords: stay cable; damper; mass; damping; frequency

1. INTRODUCTION

For a cable-stayed bridge, the damping and frequency in stay cables are very low, and thus the cables are often vulnerable to environmental excitations such as wind, wind-rain [1,2] or parametric excitations. Therefore, it is of great interest to understand the dynamic behavior of cables for possible engineering application in bridge structures. Practical measures have been taken to mitigate stay cable vibrations. One of the most widely-used damping devices is the passive viscous damper that is often installed near the cable anchorage [3, 4, 5]. The method of optimizing the damper attached to a stay cable has been comprehensively studied with consideration given to different aspects of non-ideal factors, such as damper nonlinearity, supporter flexibility, internal stiffness,

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cable sag and bending stiffness [6-9]. However, little attention has been paid to more general results involving a taut cable with a damper and mass. The damper and the clamping to connect damper and cable, or some other devices connected to the cable, are concentrate masses themselves, which will, in turn, inevitably change the dynamic behavior of the cable-damper system. There are only limited results concerning this issue. Krenk [10] and Duan [11] discussed this problem for the case of small mass values: when a mass is at the same position with viscous damper near the cable anchorage, the effect of the mass on the maximum damping will increase and that on the corresponding optimal damping constant will decrease. In this paper, the damping and the frequency of a taut cable with a damper and a mass are investigated. The approximate solution obtained for the cable-damper-mass system can facilitate the design of the damping device with a nearby concentrate mass attached to a taut cable.

2.GENERAL PROBLEM FORMULATIONS

Figure 1 shows a damper and a mass attached separately to a taut cable. *c* is the viscous damping coefficient of the damper, *M* is the concentrate mass. The length between the left cable anchorage and the damper is l_1 denoted as, whereas the length between the damper and the mass is l_2, and the length between the right cable anchorage and the mass is $l_3=L- l_1- l_2$ where *L* is the total length of the cable.

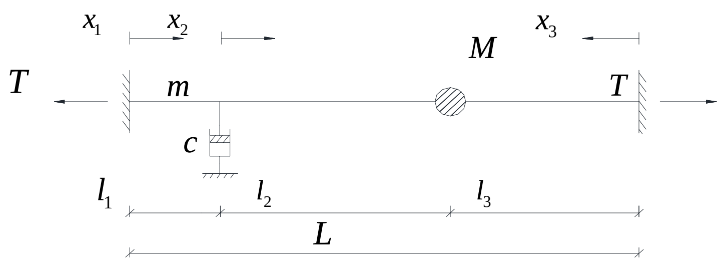

Figure 1. Taut cable with a damper and mass

Since the internal structural damping of a stay cable is very small, it is neglected in this study. The free vibration of the cable-damper-mass system in the transverse direction can be described by the following partial differential equation for each part of the cable[12]:

$$m\frac{\partial^2 y_p(x_p,t)}{\partial t^2}=T\frac{\partial^2 y_p(x_p,t)}{\partial x_p^2} \qquad (1)$$

where $y_p(x_p,t)$ is the transverse displacement of the cable at point x_p, and x_p is the coordinate along the cable chord axis in the p^{th} part $(p=1,2,3)$; *m* is the cable mass per unit length; and *T* is the cable tension force. To solve Eq. (1) subjected to boundary, continuity, and equilibrium conditions, distinct solutions over the three cable segments are assumed to exist of the form [4, 5]:

$$y_p(x_p, \tau) = Y_p(x_p)\ e^{\lambda\tau} \qquad (2)$$

where the non-dimensional time $\tau = \omega_{01} t$ and $\omega_{01} = \pi/L\sqrt{T/m}$; λ is a dimensionless eigenvalue that is complex in general，$\lambda = \alpha + \beta i$， $i = \sqrt{-1}$. For specific value of *c*，*M*，l_1/L and l_2/L，λ can be derived, which contains the information of vibration damping and frequency:

$$\lambda = \frac{\omega}{\omega_{01}}(-\xi + i\sqrt{1-\xi^2}) \qquad (3a)$$

$$\alpha = -\frac{\omega}{\omega_{01}}\xi \qquad (3b)$$

$$\beta = \frac{\omega}{\omega_{01}}\sqrt{1-\xi^2} \qquad (3c)$$

where ζ is modal damping ratio and ω is the modulus of the dimensional eigenvalue. By taking Eq. (2) into Eq. (1) the following equation can be obtained:

$$\frac{d^2 Y_p(x_p)}{dx_p^2} = \left(\frac{\pi\lambda}{L}\right)^2 Y_p(x_p) \qquad (4)$$

Because λ is complex in general, the solutions to Eq. (4), which are the mode shapes of the system, are also complex. Suppose that the mode shape could be expressed as [4, 5]:

$$Y_p(x_p) = A_p \sinh(\pi\lambda x_p/L)\sinh^{-1}(\mu_p\lambda) + B_p \cosh(\pi\lambda x_p/\mathrm{L})\cosh^{-1}(\mu_p\lambda) \qquad (5)$$

where A_p and B_p are the components of the complex amplitude, and $\mu_p=\pi l_p/L$.
Boundary, compatibility and equilibrium conditions should be satisfied for Eq. (5). For the boundary and compatibility conditions:

$$y_p\left(x_p = 0, \tau\right) = 0 \quad \text{when} \quad p = 1,3 \qquad (6a)$$

$$y_1(l_1, \tau) = y_2(0, \tau) \qquad (6b)$$

$$y_2(l_2, \tau) = y_3(l_3, \tau) \qquad (6c)$$

At the damper and the mass location, there is a discontinuity in the cable slope, providing a transverse force matching the damper and inertial force:

$$T\left(\frac{\partial y_2}{\partial x_2}\Big|_{x_2=0} - \frac{\partial y_1}{\partial x_1}\Big|_{x_1=l_1}\right) = c\frac{\partial y_1}{\partial t}\Big|_{x_1=l_1} \qquad (6d)$$

$$T\left(-\frac{\partial y_2}{\partial x_2}\Big|_{x_2=l_2} - \frac{\partial y_3}{\partial x_3}\Big|_{x_3=l_3}\right) = M\frac{\partial^2 y_2}{\partial x_2{}^2}\Big|_{x_2=l_2} \qquad (6e)$$

By substituting Eq. (2) and Eq. (4) for Eq. (6) the following equations are derived

$$B_1 = B_3 = 0 \tag{7a}$$

$$A_1 - B_2 \,\mathrm{sech}(\mu_2 \lambda) = 0 \tag{7b}$$

$$A_2 + B_2 - A_3 = 0 \tag{7c}$$

$$\left[\eta + \coth(\mu_1 \lambda)\right] A_1 - \mathrm{csch}\left(\mu_2 \lambda\right) A_2 = 0 \tag{7d}$$

$$\left[\phi + \lambda \coth\left(\pi \lambda l_2 / L\right)\right] A_2 + \left[\phi + \lambda \tanh\left(\pi \lambda l_2 / L\right)\right] B_2 + \lambda \coth\left(\pi \lambda l_3 / L\right) A_3 = 0 \tag{7e}$$

where the non-dimensional mass coefficient is *φ=πM/(mL)*, and the non-dimensional damping constant is $\eta = c\big/\sqrt{Tm}$.

The above-mentioned equations can be transformed into matrix form:

$$\mathbf{C}\Phi = 0 \tag{8}$$

where C is the complex matrix:

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & -\mathrm{sech}(\Gamma_2) & 0 \\ 0 & 1 & 1 & -1 \\ \eta + \coth(\Gamma_1) & -\mathbf{csch}(\Gamma_2) & 0 & 0 \\ 0 & \phi + \lambda \coth\left(\Gamma_2\right) & \phi + \lambda \tanh\left(\Gamma_2\right) & \lambda \coth\left(\Gamma_3\right) \end{bmatrix} \tag{9}$$

where *Γp=πλlp/L* and Φ is the corresponding complex amplitude vector:

$$\Phi = \left[A_1 \; A_2 \; B_2 \; A_3\right]^T \tag{10}$$

The infinite set of nontrivial solutions (*Φ≠0*) means that *det(Φ)=0* can be solved for the normalized frequency *λ*, and the characteristic polynomial is:

$$\begin{aligned} &\sinh\left(\Gamma\right) + \eta \sinh(\Gamma_1)\sinh(\Gamma_2 + \Gamma_3) \\ &\quad + \phi\lambda \sinh(\Gamma_1 + \Gamma_2)\sinh(\Gamma_3) + \eta\phi\lambda \sinh(\Gamma_1)\sinh(\Gamma_2)\sinh(\Gamma_3) = 0 \end{aligned} \tag{11}$$

Eq. (11) can be further re-written in terms of the imaginary- and real-part equation.

3. ASYMPTOTIC SOLUTIONS

3.1 Both damper and mass close to cable end

In case that both damper and mass are located near the cable end, and the concentrate mass is much smaller than that of the total cable mass *mL*, the frequency change induced by damper and the concentrate mass is small. This is similar to the case of a taut cable with an attached damper near a cable end. The asymptotic relationships can be found in the presence of both damper and mass. In the following sections four different cases are discussed on the basis of different damper and mass locations.

(1) When damper and mass are located near the left side anchorage, $l_1L^{-1}<<1$ and $l_2L^{-1}<<1$; meanwhile, the change in λ is small, the change in α and β is also small, and $\beta \cong \mathrm{n}+\delta\beta$. The real part and the imaginary part in terms of Taylor series can be further re-written, after some simplification becoming:

$$\frac{\xi_n}{l_1/L} \cong \frac{1}{\left(1\text{-}\mu_2\phi n^2\right)\left[1\text{-}(\mu_1+\mu_2)\phi n^2\right]} \frac{E_{ml}\pi^2\kappa_l}{1+\left(E_{ml}\pi^2\kappa_l\right)^2} \tag{12}$$

where the non-dimensional damper parameter grouping is:

$$\kappa_l = \pi^{-1} n\,\eta l_1/L \tag{13}$$

Also, the factor that considers effects of mass is:

$$E_{ml} = \frac{1\text{-}\phi n^2\mu_2}{1\text{-}\phi n^2(\mu_1+\mu_2)} = 1 + \frac{\phi n^2 \mu_1}{1\text{-}(\mu_1+\mu_2)\phi n^2} \tag{14}$$

The maximum damping and the corresponding non-dimensional damper parameter grouping could be easily derived from Eq. (12):

$$\xi_{n,max}\big/\left(l_1/L\right) \cong \frac{1}{2}\frac{1}{\left(1\text{-}\phi n^2\mu_2\right)\left[1\text{-}\phi n^2(\mu_1+\mu_2)\right]} \tag{15}$$

$$\kappa_l^{opt} \cong 1\big/\left(\pi^2 E_{ml}\right) \tag{16}$$

The first five modes of the non-dimensional damping vs. damping parameter grouping based on the numerical solution are displayed in Figure 2, together with the related approximate curves for the case $l_1/L=l_2/L=0.02$ and $\Phi=0.1$. It shows that both numerical and approximate solutions agree reasonably well for the first 4 modes, except for the 5th mode, which will be discussed later.

(2) When damper and the concentrate mass are located near the right side anchorage, $l_3L^{-1}<<1$, $l_2L^{-1}<<1$, and after the same simplification the following equations are obtained:

$$\frac{\xi_n}{\left(l_3+l_2\right)/L} \cong E_{mr}\frac{E_{mr}\kappa_r\pi^2}{1+\left(E_{mr}\kappa_r\pi^2\right)^2} \tag{17}$$

$$\kappa_r = \pi^{-1} n\eta \left(l_3 + l_2\right) / L \quad (18)$$

$$E_{mr} = \frac{1}{1-\phi\mu_3 n^2} = 1 + \frac{\phi\mu_3 n^2}{1-\phi\mu_3 n^2} \quad (19)$$

The corresponding maximum damping ratio and the optimum damping parameter grouping can be derived from Eq. (17):

$$\xi_{n,max} / \left[\left(l_3 + l_2\right)/L\right] \cong \frac{1}{2} E_{mr} \quad (20)$$

$$\kappa_r^{opt} \cong 1 / \left(\pi^2 E_{mr}\right) \quad (21)$$

Eqs. (12) and (17) can be regarded as the design formula of a cable with a concentrate mass near the viscous damper. Because the factors $\frac{1}{\left(1\text{-}\phi n^2 \mu_2\right)\left[1\text{-}\phi n^2 (\mu_1 + \mu_2)\right]}$ E_{ml} and E_{my} are larger than 1 when the product Φn^2 is smaller than 1.0 (equal to 1 only when $\Phi=0$), It can be concluded that the attainable maximum damping $\xi_{n,max}$ will be larger than in the case without a concentrate mass, and the corresponding optimum damping constant will be decreased, regardless of the type of relative location of damper and mass.

(3) A special case is that both mass and damper are connected to the cable at the same location, as was also discussed by Krenk and Høgsberg [10]. This corresponds to $l_2=0$ in Eq. (12) and (17):

$$\frac{\xi_n}{l_1/L} \cong \frac{E_{mp}\pi^2\kappa_l}{1+\left(E_{mp}\pi^2\kappa_l\right)^2} \quad (22)$$

where

$$E_{mp} = \frac{1}{1-\mu_1\phi n^2} \quad (23)$$

The corresponding maximum damping ratio $\xi_{n,max} / \left(l_1/L\right) \cong \frac{1}{2} E_{mp}$, which clearly demonstrates the increment of damping.

(4) When damper and mass are located by the two different cable ends, it is found that damper and mass do not affect each other. Similar behavior was observed for the case of a taut cable with attached damper and spring located at two different cable ends [17]. Thus it will not be further discussed in this paper.

It should be noted that the above-mentioned approximate formulas are based on the assumption that Φn^2 is far smaller than 1. Studies reveals that as Φn^2 increases, the discrepancy between the approximate prediction and the numerical solution becomes larger. The same tendency is also illustrated in Figure 2 for the fifth mode with $\Phi n^2=0.25$.

Fig. 3 shows the numerically calculated contour map of the 1st mode damping with $l_1/L=0.02$ and $l_2/L=0$, where the label in the curves indicates the value of $\xi_1 L/l_1$. It can be clearly seen that the damping firstly increases for small Φ ($n=1$ for the 1st mode), until Φ is about 15-20, at which a maximum value of $\xi_1 L/l_1$ is reached. Afterwards, $\xi_1 L/l_1$ decreases sharply. The Eq. (23) roughly predicts this critical non-dimension mass, i.e. $\Phi=1/\mu_1=15.92$, which is illustrated in Fig. 3 for a range from about 15 to 18.

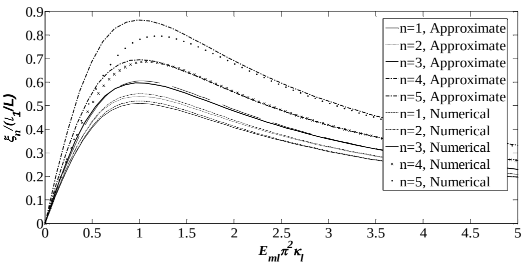

Figure 2. Comparison of approximate results with numerical solutions ($l_1/L=l_2/L=0.02$, $\Phi=0.1$)

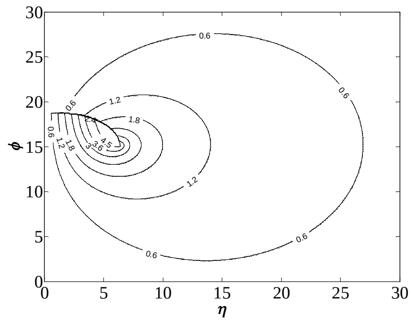

Figure 3. Contour maps of 1st mode damping ($l_1/L= 0.02$, $l_2/L=0$)

4. CONCLUSIONS

The damping and frequency of a taut cable attached with a viscous damper and mass is analyzed in this paper. The effects of the non-dimensional mass coefficient and the mass location on cable vibration, damping, and frequency are dealt with in detail. The concentrate mass increases the maximum attainable cable vibration damping when a damper and a mass are both placed near the cable anchorage and the non-dimensional mass coefficient is smaller than some critical value. However, the mass will significantly decrease the attainable cable damping if the non-dimensional mass coefficient is larger than the critical value. The mass has negligible effects on the cable damping when a mass and a damper are placed near two different cable anchorages.

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