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Development of joint probability distribution of wind speed and direction using long-term monitoring data

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ABSTRACT

Wind loading and effects are crucial parameters in the process of structural design and safety evaluation of large-scale bridge structures. One of the most distinctive characteristics of the wind action and the wind-induced structural responses behaves as the randomness with a considerable number of uncertainties. Therefore, it is essential to develop the stochastic characterization method for effectively examining the wind load properties. In this study, the wind monitoring data measured by the ultrasonic anemometers as a part of the structural health monitoring (SHM) system instrumented on the arch Jiubao Bridge located in Hangzhou, China are extracted for development of the joint probability distribution of the wind speed and direction. An angular-linear approach is applied to construct the probability distribution function. The Weibull mixture distributions are employed to represent the wind speed distribution, and the von Mises mixture distributions are utilized to model the wind direction distribution. The expectation maximum (EM) algorithm is used to estimate the parameters of the finite mixture distributions. The Akaike's information criterion (AIC) and the distance measure are adopted to evaluate the modeling performance. The results indicate that the proposed procedure has a favorable capability in modeling the joint probability distribution of the wind speed and direction.

1. INTRODUCTION

With the rapid increase of bridge span, the surrounding wind environment becomes more important for long-span bridges constructed in wind-prone areas, especially for the bridges vulnerable to strong winds or typhoons (Fujino 2002). The typical example is the collapse of the Tacoma Narrows Bridge in 1940. However, the wind speed and wind direction are random in both time and space. Thus, it is critical to

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obtain the statistical characteristics of wind field by analyzing massive measured data. The structural health monitoring (SHM) system has been developed to evaluate the safety and durability of a structure (Ou and Li 2010). The SHM system provides an efficient approach to acquire the long-term monitoring wind data. Wang et al. (2013) analyzed the real-time wind data recorded by 3D ultrasonic anemometers included in the SHM system of Sutong Bridge and determined the wind parameters of the east coastal area of China.

Some investigations can be found on the construction of joint distribution model of the wind speed and direction. Johnson and Wehrly (1978) conducted the angular-linear (AL) distributions to model the joint distribution of bivariate random variables when one variable is directional and one is scalar. In this study, the wind data are collected by ultrasonic anemometers installed on the arch Jiubao Bridge located in Hangzhou, China. The sufficient wind data facilitate the analysis of wind field characteristics and modeling of joint distribution of wind speed and direction. The AL approach is selected to construct the joint distribution by marginal distributions of two variables, and a mixture of Weibull and von Mises distribution is chose to represent the distribution of wind speed and direction, respectively. The expectation maximum (EM) algorithm is used to estimate the parameters of marginal distributions. The optimal joint distribution model is determined by the Akaike's information criterion (AIC) and the distance measure.

2. METHODOLOGY

2.1 Angular-linear Approach

There are various approaches of constructing joint distributions by using the marginal distributions of wind speed and direction, such as Farlie-Gumbel-Morgenstern approach, angular-linear approach, and anisotropic lognormal approach (Erdem and Shi 2011). In this study, the angular-linear approach is applied to construct the bivariate joint distribution. The probability density of an angular-linear approach is defined as

$$f_{v,\theta}(v,\theta) = 2\pi g(\xi)\ f_v(v)\ f_\theta(\theta) \tag{1}$$

where $f_v(v)$ is the probability density function of the wind speed, and $f_\theta(\theta)$ is the probability density function of the wind direction.

Due to the distribution is multimodal and complex, we model the probability distributions by the method of finite mixture distributions, which is composed by two or more probability density functions. McLachlan et al. (2003) defined that the basic structure of the finite mixture distributions for independent scalar or vector observations can be defined as a weighted sum of component distributions. In this study, a two-parameter Weibull distribution is used to model the wind speed, and a mixture of von Mises distributions is used to represent the wind direction and the circular variable.

2.2 Parameter Estimation

The EM algorithm is a general method of finding the maximum-likelihood estimation of the parameters of an underlying distribution from the given data when the data are incomplete or have missing values (Ng and McLachlan 2003). The EM algorithm is widely used to estimate the parameters of finite mixture distributions. In this study, we define the finite mixture distribution model as

$$p(x|\Theta) = \sum_{i=1}^{N} w_i f_i(x|\theta_i) \tag{2}$$

where $\Theta=(\omega 1,\ldots, \omega_M,\theta_1,\ldots,\theta_M)$ are the parameters of finite mixture distribution function, and each $f_i(x)$ is the density function parameterized by θ_i.

Expanding the observation data x with the unknown quantities y, we can obtain the complete data $z=(x, y)$. The density function can be defined as

$$p(z|\ \Theta) = p(x,y|\ \Theta) = p(y|\ x,\Theta)\,p(x|\ \Theta) \tag{3}$$

Let $L(\Theta|x,y) = p(x,y|\ \Theta)$ be the joint likelihood for complete data z. After giving an initial parameter estimation Θ^0, we iterate the following steps until the results arrive the termination criterion:

E-step: Evaluate $Q(\Theta,\Theta^{(t)}) = \mathrm{E}(\log p(x,y|\Theta)|x,\Theta^{(t)})$. The unknown quantities y are obtained by the observed data x and the old parameter estimation Θ^t.

M-step: Find the $\Theta=\Theta^{t+1}$ which maximizes $Q(\Theta,\Theta^{(t)})$.

2.3 Selection of Models

In this study, we adopt the AIC and the distance measure as the fit performance to choose the optimal number of components. The AIC is a measure of the quality of statistical model (Akaike 1974), and is defined as

$$\mathrm{AIC} = 2M - 2\ln(L) \tag{4}$$

where M is the number of unknown parameters in the finite mixture distribution, and L is the value of maximum likelihood function of the mixed distribution.

The distance measure (Δc) represents the distance between the estimated and observed frequencies, and is defined as

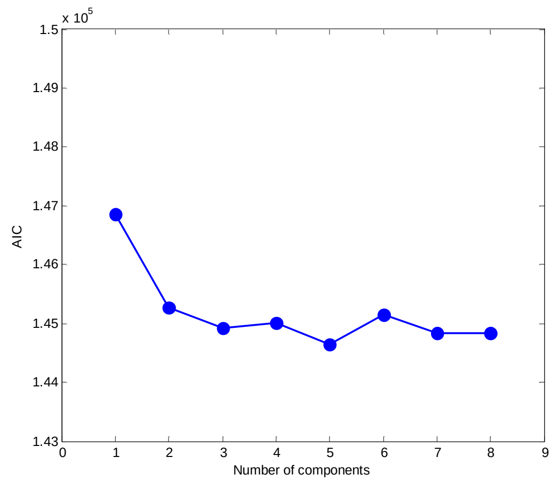
$$\Delta c = \sum_{k=1}^{m} \frac{(p_k - q_k)^2}{p_k} \tag{5}$$

where p_k and q_k represent the estimated frequency and the observed frequency, respectively, and m is the number of intervals. The preferred model is the one with the minimum values of AIC and Δc.

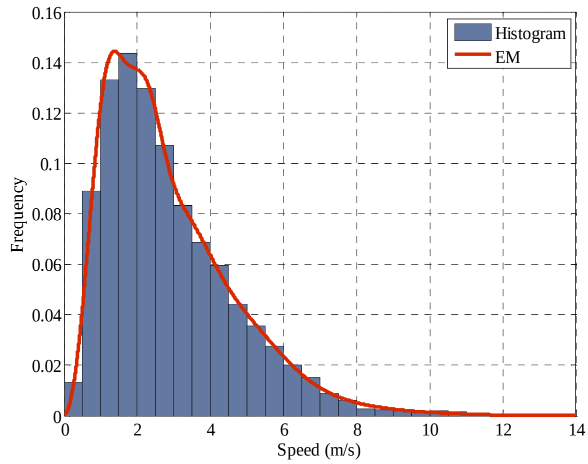
3. APPLICATION TO AN ARCH BRIDGE

In this study, the wind data recorded by the structural health monitoring (SHM) system of the arch Jiubao Bridge are analyzed to obtain the joint distribution of wind speed and direction. A one-year monitoring data from September 1, 2014 to August 31, 2015 are extracted for analysis. After obtaining 10-minute mean wind data, the EM algorithm is applied to estimate the parameters.

Fig. 1(a) shows the values of AIC with different numbers of components. It is seen from Fig. 1(a) that the AIC values of EM algorithm reach the minimum with five components. A mixture of five Weibull distributions is taken as the probability model of the wind speed, as shown in Fig. 1(b). Similarly, as shown in Fig. 2(a), the AIC values of EM algorithm reach the minimum at four components, and the mixture of four von Mises distributions is applied to represent wind direction distribution, as illustrated in Fig. 2(b). Then, we use Eq. (1) to obtain the joint distribution of wind speed and direction, as shown in Fig. 3.

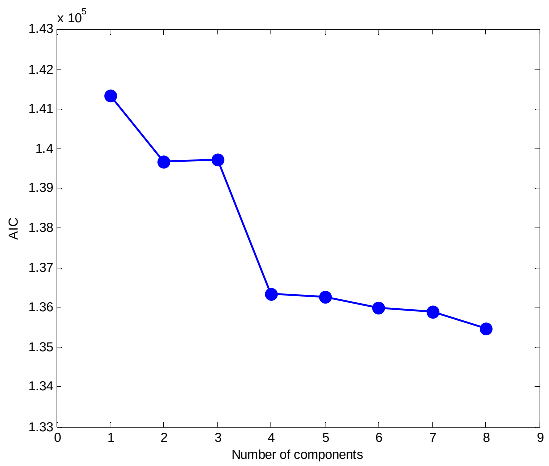


(a) AIC values with different components

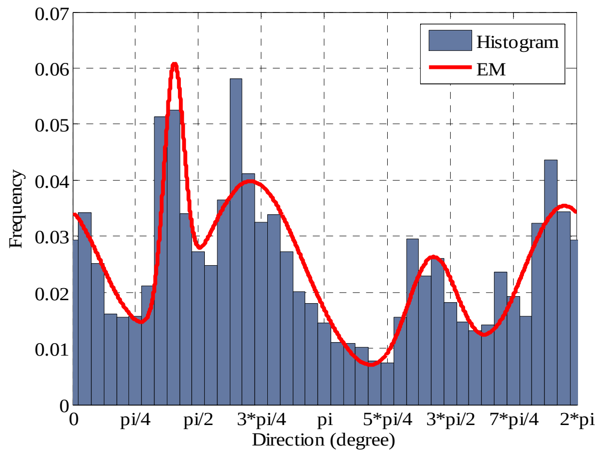


(b) Probability density function

Fig. 1 Wind speed distribution



(a) AIC values with different components



(b) Probability density function

Fig. 2 Probability distribution of wind direction

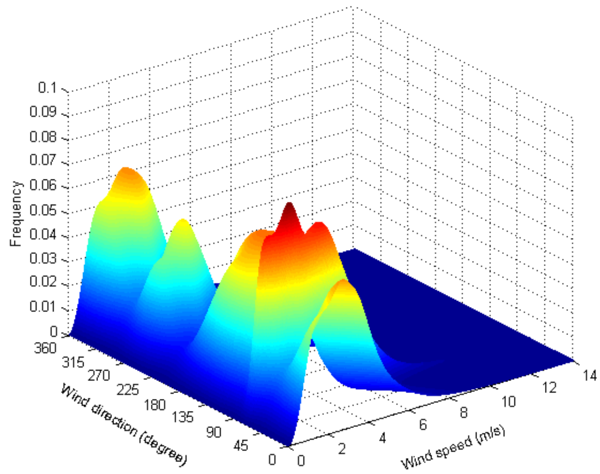


Fig. 3 Joint probability distribution of wind speed and direction

4. CONCLUSIONS

In this study, the AL approach is applied to construct the joint distribution of wind speed and direction. The EM algorithm is used to estimate the parameters of the finite mixture distribution. Through examining the values of AIC and distance measure, the optimal probability distribution model is determined. The long-term wind monitoring data are collected by the SHM system instrumented on the arch Jiubao Bridge located in Hangzhou, China, the proposed procedure for modelling the joint probability distribution model of wind speed and direction is exemplified. The wind speed and direction data are characterized by the mixture of Weibull distributions and von Mises distributions, respectively. The joint probability distribution of wind speed and direction is obtained by use of the AL approach.

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