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A Novel Vibration-Based Two-Stage Bayesian System Identification Method

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ABSTRACT

Model updating aims at making inference on the structural model parameters (e.g., stiffness, mass) based on measured structural response. Based on vibration data, two-stage approach is commonly adopted, i.e., first identify the modal parameters including natural frequencies, mode shapes, and then update the structural parameters utilizing this information by building a finite element model. Identification uncertainty in Stage I is usually not accounted for in traditional methods but this becomes critical for output-only measurement. This paper presents a novel two stages Bayesian system identification method by using the data collected under ambient condition. In Stage I, a Fast Bayesian FFT method is conducted to obtain the most probable value (MPV) of modal parameters and the associated posterior uncertainty. In Stage II, both the MPV and the posterior uncertainty obtained are formulated into the updating process to identify the model parameters. In this method, heuristics that are commonly applied in formulating the likelihood function in the traditional methods are not involved. The proposed method is illustrated by experimental data.

1. INTRODUCTION

System identification has attracted increasing attention to make inference about the parameters of a mathematical model on the basis of observed vibration data of real structures (Yuen et al. 2006; Lam et al. 2014; Yang et al. 2015). The Bayesian approach provides a fundamental means for system identification, where uncertainties due to the lack of information in the context of probability logic can be resolved (Au et al.

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2013). The parameters are viewed as uncertain variables and the identification results are cast in terms of their probability distribution after incorporating information from the observed data. This paper develops a novel two stages Bayesian system identification method by using vibration data collected under ambient condition. In Stage I, modal identification is performed by a recently developed Fast Bayesian FFT method to obtain the most probable value (MPV) of modal parameters and the associated posterior uncertainty (Au 2011, Zhang and Au 2013, Au 2012a,b). In Stage II, the MPV and the posterior uncertainty of modal parameters obtained in Stage I are both incorporated into the updating process to identify the model parameters, where heuristics commonly applied in formulating the likelihood function are not involved. Experimental data of a steel frame are used to investigate its applications. For the detail of this paper, please refer to Zhang and Au (2016).

2. STAGE I-BAYESIAN MODAL IDENTIFICATION

In Stage I, acceleration data under ambient condition is first collected. To analyze the measured data, Bayesian method is employed to perform modal identification. The basic idea is as follows. The real and imagery part of the FFT of measured data is proved to follow Gaussian distribution. According to Bayes' Theorem, the posterior probability density function (PDF) of modal parameters given the data is proportional to the likelihood function. The likelihood function can be constructed by using the properties of the FFT data. Theoretically, the modal parameters can be determined by maximizing the posterior PDF. However, if performing the optimization directly, some computational difficulties may exist in the process of optimization, for example the ill-conditioned problem and the problem of the number of modal parameters to be optimized increasing with the measured degrees of freedom (DOFs). To solve these problems and make this method well applied in practice, the cases for well-separated modes and general multiple modes are considered separately. Please refer to Au (2011), Zhang and Au (2013) and Au (2012a,b) for details. In this method, the MPV of modal parameters and the associated posterior uncertainty can be obtained directly. This makes it possible to perform model updating in Stage II by incorporating the MPV and the uncertainty information.

3. STAGE II-MODEL UPDATING

In Stage II the structural model parameters $\boldsymbol{\theta}$ will be identified by using the modal parameters obtained in Stage I. In this paper it is assumed that $\boldsymbol{\theta}$ can characterize the modal parameters ϖ (including natural frequencies and mode shapes) only through the stiffness matrix $\mathbf{K}$ and mass matrix $\mathbf{M}$.

Define $\tilde{\varpi}(\boldsymbol{\theta}) = [\tilde{\mathbf{f}}(\boldsymbol{\theta}), \tilde{\boldsymbol{\Phi}}(\boldsymbol{\theta})]$, comprising the theoretical natural frequencies $\tilde{\mathbf{f}}(\boldsymbol{\theta})$ and partial mode shapes $\tilde{\boldsymbol{\Phi}}(\boldsymbol{\theta})$, which can be determined from the eigenvalue equation for given $\boldsymbol{\theta}$. It is assumed that the natural frequencies and mode shapes can be completely determined by $\boldsymbol{\theta}$. It is shown that the posterior PDF $p(\boldsymbol{\theta} | D)$ of the structural parameters given data is expressed as,

$$p(\boldsymbol{\theta} \mid D) \propto p(\boldsymbol{\theta}) \prod_{r=1}^{n_B} \phi(\tilde{\varpi}^{(r)}(\boldsymbol{\theta}); \hat{\varpi}^{(r)}, \mathbf{C}_D^{(r)}) \propto p(\boldsymbol{\theta}) \exp[-L_{\mathrm{II}}(\boldsymbol{\theta})] \quad (0)$$

where $\phi(\tilde{\varpi}^{(r)}(\boldsymbol{\theta}); \hat{\varpi}^{(r)}, \mathbf{C}_D^{(r)})$ is a Gaussian distribution centered at the MPV $\hat{\varpi}^{(r)}$ with a covariance matrix $\mathbf{C}_D^{(r)}$; $\tilde{\varpi}^{(r)}(\boldsymbol{\theta})$ is the theoretical natural frequencies and partial mode shapes in rth band;

$$L_{\mathrm{II}}(\boldsymbol{\theta}) = \frac{1}{2} \sum_{r=1}^{n_B} [\tilde{\varpi}^{(r)}(\boldsymbol{\theta}) - \hat{\varpi}^{(r)}]^T \mathbf{C}_D^{(r)^{-1}} [\tilde{\varpi}^{(r)}(\boldsymbol{\theta}) - \hat{\varpi}^{(r)}] \quad (0)$$

is the negative log-likelihood function (NLLF), which can transfer the maximum problem to the minimizing problem. The form of $L_{\mathrm{II}}(\boldsymbol{\theta})$ resembles some measure-of-fit function between the MPV $\hat{\varpi}$ and its model counterpart $\tilde{\varpi}(\boldsymbol{\theta})$. The quadratic nature of the discrepancy $[\tilde{\varpi}^{(r)}(\boldsymbol{\theta}) - \hat{\varpi}^{(r)}]$ stems from the Gaussian nature of the posterior distribution of modal parameters in the first stage.

Based on Eq. (2), the model parameters can be obtained directly by minimizing the NLLF if it is globally identifiable. The posterior uncertainty of model parameters can also be evaluated by the finite difference method. For the details of this method, please refer to Zhang and Au (2016). Note that the case considered here reflects that the natural frequencies and mode shapes in Stage I are directly related to the structural model parameters with structural modeling error not considered. In the general theory developed by the same authors presents a fundamental means for taking into account the structural modelling error (Au and Zhang 2016). This modeling is directly related to the fidelity of the structural model under question and will be investigated in the future work.

4. APPLICATION IN EXPERIMENTAL DATA

A three-storied shear frame model situated in the structural heavy laboratory at City University of Hong Kong is used to illustrate the proposed method. The dimension information of this structure is known. Assuming an initial elastic modulus of $1.55 \times 10^{11} N/m^2$, the interstory stiffness along the weak direction is 3.07 N/mm, 10.41 N/mm and 11.33 N/mm for 1st to 3rd floor, respectively and interstory stiffness along strong direction is 76.69 N/mm, 260.18 N/mm and 283.34 N/mm for 1st to 3rd floor, respectively.

4.1 Stage I-Modal identification

At the four corners of each floor, bi-axial sensors were instrumented to measure the structure response along the translational directions, with 24 measured degrees of freedom in all the three floors. Ten minutes data were measured acquired with a sampling rate of 2048Hz. The modal identification results are shown in
Table 1. The Mode shape is ignored here. Nine modes were identified including three translational modes in X direction (denoted by TX), three translational modes in Y direction (denoted by TY) and three torsional modes (denoted by R). The natural

frequencies ranged from 0.81 Hz to 45.86 Hz. The damping ratios are small and they are all less than 0.5%. The COVs (Coefficient of Variation=Standard derivation / MPV) of natural frequency are quite small in the order of magnitude of less than 0.1%. This may be attributed to the higher signal to noise ratio during the measurement. The COV of damping ratio is much larger than that of natural frequency and it is obviously observed that the COV tends to decrease with the increase of mode numbers.

Table 1: Identified modal parameters

Mode	Nature	Frequency MPV (Hz)	Frequency COV (%)	Damping ratio MPV (%)	Damping ratio COV (%)
1st	TX1	0.81	0.09	0.3	39
2nd	TX2	3.52	0.03	0.1	26
3rd	TY1	4.82	0.02	0.1	30
4th	TX3	5.90	0.02	0.1	21
5th	R1	7.11	0.02	0.2	16
6th	TY2	17.66	0.01	0.1	11
7th	R2	26.37	0.02	0.3	6
8th	TY3	31.49	0.01	0.1	11
9th	R3	45.86	0.01	0.1	7

4.2 Stage II-Structural model identification

In this stage, the mass properties are assumed to be known, and so ignore the mass of the columns. Thus only the stiffness of the columns is unknown and will be identified. Since the dimensions of the four columns in each floor are similar, the lateral stiffness of the four columns in each story is assumed to be identical and they are equal to a quarter of the total interstory stiffness. For the i-th story ($i = 1,2,3$), the interstory stiffness along the x and y directions are parameterized by

$$\begin{aligned} k_{xi} &= \theta_{xi} k'_{xi} \\ k_{yi} &= \theta_{yi} k'_{yi} \end{aligned} \qquad (0)$$

where k'_{xi} and k'_{yi} are the nominal value of interstory stiffness along the x and y directions, respectively; and

$$\boldsymbol{\theta} = [\theta_{x1}, \theta_{x2}, \theta_{x3}, \theta_{y_1}, \theta_{y_2}, \theta_{y3}] \qquad (0)$$

denotes one set of dimensionless structural model parameters to be identified.

All the data collected from measured DOFs are used for the first stage modal identification and second stage model updating. Table 2 shows the identification results for the MPV of structural model parameters and the associated posterior uncertainty in Stage II. The MPVs of model parameters are generally close to 1 with a small posterior uncertainty except for θ_{x1}. This shows that the actual stiffness properties are similar to the calculated ones on the basis of the available nominal information. The abnormally low value of θ_{x1} may be because of the reduction in stiffness of this floor due to a hole existing in each column. This has been ignored in the calculation of the nominal stiffness. The posterior COV of the stiffness parameters are generally small. This may be due to the small posterior uncertainty of the modal parameters obtained in the first stage.

Table 2: Identified stiffness parameters (Stage II)

Story i	θ_{xi} MPV (COV)	θ_{yi} MPV (COV)
1st	0.700 (0.21%)	1.015 (0.03%)
2nd	1.011 (0.08%)	1.183 (0.06%)
3rd	1.078 (0.07%)	1.118 (0.06%)

CONCLUSIONS

This paper presents a two-stage Bayesian system identification method. This method can incorporate the information of both the MPV of modal parameters and the associated posterior uncertainty. The accuracy of identified modal parameters in the Stage I can be considered in the Stage II model updating. This is quite meaningful for the vibration data collected under ambient condition since it is output-only data and the loading is random, leading to possibly large posterior uncertainty for the modal parameters obtained. A steel frame structure in the laboratory is used to illustrate the proposed method. By the investigation, it is shown that the proposed method can identify reasonable model parameters and the associated posterior uncertainty is also small.

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