

## **Subspace flexibility identification from multiple reference impact test data**

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### **ABSTRACT**

Impact testing is an effective means of identifying structural flexibility. However, most related flexibility identification methods have strict requirements on the type of force that provides the impact. This is because most methods operate in the frequency domain, which can give incorrect flexibility identification results when double or multiple clicks occur in an impact test. This article proposes a method to estimate structural modal scaling coefficients and flexibility characteristics using a subspace identification algorithm in the time domain. The advantage of the proposed method is that it adapts to the input force type thus it is potential to be widely used in engineering practices. Numerical and experimental examples are presented to illustrate the effectiveness and robustness of the proposed method.

**Keywords:** Impact test, flexibility, subspace identification, impacting force type, robustness.

### **1 INTRODUCTION**

Structural identification is used to identify unobservable responses or attributes of a structure (e.g., load carrying capacity, seismic vulnerability) from observable structural responses (e.g., strains, displacements). The technology underlying structural identification has been widely investigated (Adeli and Jiang, 2006; Adeli and Jiang, 2009; Marano et al., 2011; Raich and Liskai, 2012; Guo and Karreem, 2015; Yuen and Mu, 2015). Operational modal analysis aims to identify structural modal properties from ambient vibration data, and has been widely applied for the modal identification of civil structures (Reynders, 2012; Yan and Ren, 2012). Its limitation is that the magnitudes of the structural frequency response functions (FRFs) estimated from ambient vibration data are incorrect, because the input force is unknown (Zhang et al., 2014). Thus, it is

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difficult to identify deep-level structural parameters such as flexibility.

Impact testing has the advantage of estimating exact structural FRFs, including their magnitudes, by measuring both input and output data, giving it the potential to achieve better identification results than ambient vibration tests. With the development of large and powerful impacting devices, the impact testing of civil structures, especially small and medium span bridges, is of increasing interest to researchers and engineers. Multiple-reference impact testing has been developed to identify parameters such as the modal scaling factors (Raghavendracher and Aktan, 1992). Brownjohn et al. (2010) used a shaker to excite the Wetteren footbridge for structural dynamical testing and modal identification. Zhang et al. (2015) proposed the rapid impact test methodology by developing a vehicle equipped with a mobile impacting device for convenient structural vibration tests. One of the merits of the impact test is that it can estimate the FRF magnitudes, from which the flexibility (defined as the inverse of structural stiffness) can be identified. Catbas et al. (2004) developed the complex mode indicator function (CMIF) method to determine the modal properties with proper scaling for structural flexibility identification. Zhang et al. (2012) proposed a Sub-PolyMAX method to identify structural flexibility by implementing a least-squares solver in each subspace of the whole frequency range. These approaches have the common feature that the identification procedure is performed in the frequency domain. Thus, their results largely rely on the accuracy of the estimated FRFs. It is known that the accuracy of these estimated FRFs is greatly influenced by the input force type. Double or multiple clicks in the impact test are generally thought to be unacceptable, because they generate an abnormal force spectrum. Furthermore, the leakage problem frequently occurs when processing the field test data in the frequency domain, as measured structural responses often do not decay freely to zero because of the existence of other uncontrolled forces such as traffic and wind. The above challenging problems greatly limit the application of frequency-domain structural flexibility identification methods in engineering practices.

In this article, a time domain subspace flexibility identification technology is proposed. It has the merit of adapting to the input force type. The subspace identification technology is classified as either a deterministic or stochastic subspace method, depending on whether the input force is used or not. Data-driven and covariance-driven algorithms for solving the subspace identification problem have been presented (Overschee and Moor, 1996; Carden and Brownjohn, 2008; Kurka and Cambraia, 2008; Gandino et al., 2013; Sun et al., 2015). In this article, a subspace identification technology is developed that identifies structural flexibility using the modal scaling factors calculated by directly decoupling the complete state matrix. Because the proposed method does not need to estimate the structure FRFs, which are sensitive to the input force type and suffer from the leakage problem, it is applicable to various types of input force. Thus, the proposed method has the potential to be used by many engineers for a wide range of applications.

The structure of this article is as follows. The subspace flexibility identification technology performed in the time domain is presented in Section 2. This includes the estimation of the system matrices of the state-space equation, calculating structural modal scaling factors, and identifying structure flexibility characteristics. A numerical example of a reinforced concrete bridge and an experimental example of a simple supported beam are studied in Sections 3 and 4, respectively, to verify the proposed

method and illustrate its superiority over frequency domain methods. Conclusions are drawn in Section 5.

## 2 SUBSPACE FLEXIBILITY IDENTIFICATION METHOD

### 2.1 Basic modal parameter identification

The discrete-time state-space model of a structure is

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + w_k \\ y_k = Cx_k + Du_k + v_k \end{cases} \quad (1)$$

Where  $x_k, u_k, y_k$  are the state vector, the input vector and the output vector at time instant  $k$ , respectively.  $A, B, C, D$  are the system matrices, and  $w_k, v_k$  denote the process noise due to disturbances and modeling inaccuracies and the measurement noise due to sensor inaccuracy, respectively.  $w_k$  and  $v_k$  are assumed to be white noise.

A reference-based data-driven subspace identification algorithm is used to solve the system matrices in Eq. (1) from input and output data (Reynders and Roeck, 2008). First, we gather the output data in a block Hankel matrix with  $2i$  block rows and  $j$  columns:

$$Y_{0/2i-1} = \begin{bmatrix} y_0^{ref} & y_1^{ref} & y_2^{ref} & \dots & y_{j-1}^{ref} \\ y_1^{ref} & y_2^{ref} & y_3^{ref} & \dots & y_j^{ref} \\ \dots & \dots & \dots & \dots & \dots \\ y_{i-1}^{ref} & y_i^{ref} & y_{i+1}^{ref} & \dots & y_{i+j-2}^{ref} \\ \hline y_i & y_{i+1} & y_{i+2} & \dots & y_{i+j-1} \\ y_{i+1} & y_{i+2} & y_{i+3} & \dots & y_{i+j} \\ \dots & \dots & \dots & \dots & \dots \\ y_{2i-1} & y_{2i} & y_{2i+1} & \dots & y_{2i+j-2} \end{bmatrix} = \begin{bmatrix} Y_p^{ref} \\ Y_f \end{bmatrix} \quad (2)$$

The subscripts of  $Y_{0/2i-1}$  denote the subscripts of the first and last element in the first column of the block Hankel matrix, the subscripts  $p$  and  $f$  denote past and future, and  $y_k^{ref}$  are the  $k^{\text{th}}$  reference outputs.. Similarly, the input block Hankel matrices  $U_p$  and  $U_f$  are generated. The projection operation is performed as shown below.

$$O_i = Y_f / U_f \begin{bmatrix} U_p \\ Y_p^{ref} \end{bmatrix}; \quad (3)$$

$$L_i = Y_f / \begin{bmatrix} U_p \\ Y_p^{ref} \\ U_f \end{bmatrix}; \quad L_{i+1} = Y_{i+1/2i-1} / \begin{bmatrix} U_{0/i} \\ Y_{0/i}^{ref} \\ U_{i+1/2i-1} \end{bmatrix} \quad (4)$$

where  $O_i$  is the oblique projection of the row space of  $Y_f$  on the joint row space of  $U_p$  and  $Y_p^{ref}$  in the direction of the row space of  $U_f$ .  $L_i, L_{i+1}$  are the orthogonal projection. The notation for input and output block Hankel matrices in Eq. (4) are similar as in Eq. (2).

The extended observability matrix  $\Gamma_i$  is calculated by performing singular value decomposition,

$$W_1 O_i W_2 = [U_1 \quad U_2] \begin{bmatrix} S_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} = U_1 S_1 V_1^T \quad (5)$$

$$\Gamma_i = W_1^{-1} U_1 S_1^{1/2} \quad (6)$$

$W_1$  and  $W_2$  are user-defined weighting matrices. The matrices  $A$ ,  $C$ , and  $K_f$  can then be obtained from the least-squares solution of the equation:

$$\begin{bmatrix} \Gamma_{i-1}^\dagger L_{i+1} \\ Y_{i/i} \end{bmatrix} = \begin{bmatrix} A \\ C \end{bmatrix} \Gamma_i^\dagger L_i + K_f U_f + \begin{bmatrix} \rho_{wi} \\ \rho_{vi} \end{bmatrix} \quad (7)$$

Where  $\bullet^\dagger$  denotes the Moore-Penrose pseudo-inverse of the matrix  $\bullet$ .  $\Gamma_{i-1}$  denotes the matrix  $\Gamma_i$  without the last element.  $Y_{i/i}$  is the  $i^{\text{th}}$  row of  $Y_{0/2i-1}$ .  $\rho_{wi}$ ,  $\rho_{vi}$  are error terms corresponding to  $w_k$  and  $v_k$ . The system matrices  $B$ , and  $D$  can be determined by

$$K_{fk} = N_k [D^T \quad B^T]^T \quad (8)$$

Where  $K_{fk}$  is the  $k^{\text{th}}$  column of  $K_f$ , and  $N_k$  is generated through the recombination of known matrix. The details of solving the system matrices of the state-space equation were given by Overschee and Moor (1996).

Once the system matrices of the state-space equation have been determined from Eqs. (7) and (8), the structural basic modal parameters can be identified. The eigenvalue decomposition of  $A$  gives  $A\Psi = \Psi\Lambda$ , where  $\Lambda = \text{diag}(\lambda_i)$  is a diagonal matrix, and the diagonal elements  $\lambda_i$  is the eigenvalue from which the structural frequencies and damping ratios are identified.  $\Psi$  is the eigenvector from which the mode shapes are identified through the relation  $\phi_i = C\Psi$ .

## 2.2 Modal scaling estimation and flexibility identification

Current subspace identification methods only identify the basic modal parameters, such as frequency, damping ratio, and mode shapes. A subspace method to identify structural flexibility is proposed and presented below.

The discrete-time state-space equation can be rewritten as

$$\begin{cases} x_{k+1}^m = \Lambda x_k^m + B^m u_k + w_k \\ y_k = C^m x_k^m + D u_k + v_k \end{cases} \quad (9)$$

where  $A = \Psi\Lambda\Psi^{-1}$ ,  $x_k^m = \Psi^{-1}x_k$ ,  $B^m = \Psi^{-1}B$ , and  $C^m = C\Psi$ . Taking the Fourier transform of Eq. (13), the FRFs of the structure can be written in the following form:

$$H(w) = C^m ((zI - \Lambda)^{-1} + (\Lambda - I)^{-1}) B^m, \quad z = e^{jw\Delta t} \quad (10)$$

When the measured output  $y$  is acceleration,  $H(w)$  denotes the structural acceleration FRFs  $H_a(w)$ . From Eq. (10), the structure FRFs can be synthesized from the identified system matrices of the state-space equation. The synthesized acceleration FRFs  $H_a(w)$  given by Eq. (10) can be expressed in the following form:

$$H_a(w) = \sum_{i=1}^n \frac{(z-1)\phi_i b_i^m}{(\lambda_i-1)(z-\lambda_i)} \quad (11)$$

where  $\phi_i$  is the  $i^{\text{th}}$  mode shape of the structure, and  $b_i^m$  is the  $i^{\text{th}}$  row of  $B^m$ .

It is known that the displacement FRFs can be rewritten in the form:

$$H_d(w) = \sum_{i=1}^{n/2} \left( \frac{q_i \phi_i \phi_i^T}{jw - \lambda_{ci}} + \frac{\bar{q}_i \bar{\phi}_i \bar{\phi}_i^T}{jw - \bar{\lambda}_{ci}} \right) \quad (12)$$

where  $H_d$  denotes the displacement FRFs,  $q_i$  is defined as the modal scaling factor of the  $i^{\text{th}}$  mode,  $\bar{\bullet}$  denotes the complex conjugate, and  $\bullet^T$  denotes the Hermitian transpose.

The acceleration FRFs and displacement FRFs are related as follows:  $H_a(w) = -w^2 H_d(w)$ . From Eqs. (11) and (12), we can derive that

$$-\frac{w^2 q_i \phi_i \phi_i^T}{j\omega - \lambda_{ci}} = \frac{(z-1)\phi_i b_i^m}{(\lambda_i - 1)(z - \lambda_i)} \quad (13)$$

Therefore, the modal scaling factor  $q_i$  is obtained from Eq. (13). The flexibility is then identified by

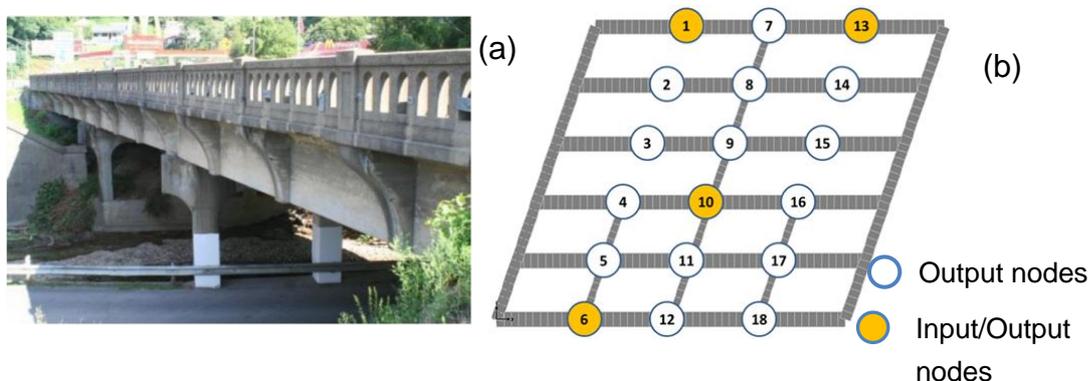
$$f = H_d(0) = \sum_{i=1}^{n/2} \left( \frac{q_i \phi_i \phi_i^T}{-\lambda_{ci}} + \frac{\bar{q}_i \bar{\phi}_i \bar{\phi}_i^T}{-\bar{\lambda}_{ci}} \right) \quad (14)$$

The above procedure shows that the proposed method identifies structural modal scaling factors by decoupling the synthesized FRFs, and then identifies structural flexibility from multiple reference impact test data. In the proposed method, structural FRFs are synthesized through Eq. (10) using the system matrices identified by the subspace algorithm in the time domain. Thus, our approach avoids the leakage problem and does not strictly require the input force type needed by frequency domain methods.

### 3 EXAMPLE STUDY OF A REINFORCED CONCRETE BRIDGE

#### 3.1 Example description

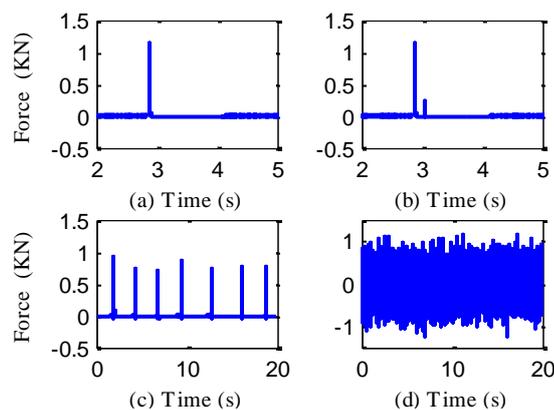
A numerical example of a reinforced concrete bridge is first studied to verify the robustness of the proposed method to various excitations (Figure 1(a)). The prototype finite element (FE) model is a three-span T-beam bridge, a simple-supported reinforced concrete structure with a skew of approximately  $18^\circ$ . Each span of the bridge is 14.6 m long; with a width along the skew of 14.6 m. The FE model of the bridge was constructed in the SAP2000 software. The beams and pier caps are connected using rigid links, forcing the components to act compositely. The support conditions at the abutments are pins, while the base of the piers is fixed. In total, the model comprises 1408 frame elements and 336 rigid links. Because the three spans of the bridge are very weakly coupled, each span can be considered as an individual structure. To illustrate the procedure of the proposed method, the left span with the sensor layout shown in Figure 1(b) is studied. Eighteen nodes are selected as output nodes ( $N_o = 18$ ), and their structural accelerations under each impact are recorded. The four input nodes ( $N_i = 4$ ) shown in Figure 1(b) are selected as the excitation locations in the multiple reference impact test. Namely, the excitation force is applied on the input nodes in turn until all input nodes have been impacted.



**Fig. 1.** Bridge model: (a) bridge prototype; (b) impact test sensor layout.

As discussed earlier, the advantage of the proposed method is its robustness to

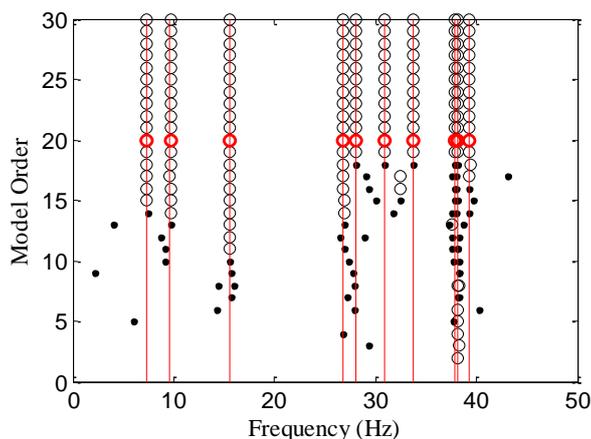
different input force types. To verify and illustrate this feature, the cases of single impacting, double impacting, multiple impacting, and random excitations were studied, as shown in Figure 2. The first three forces are from a bridge field test using sledgehammers, and the fourth is a randomly generated white noise. The CMIF method, which is a representative flexibility identification technique in the frequency domain, was also performed, and the results are compared for verification.



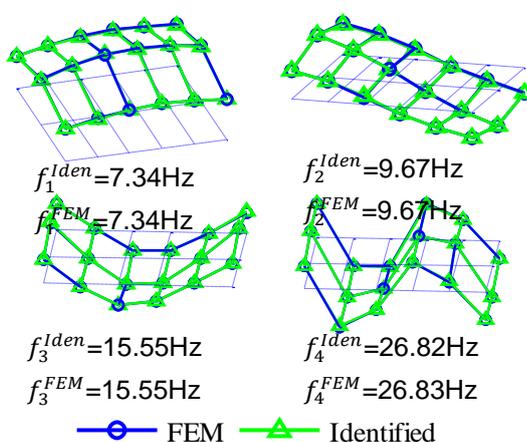
**Fig. 2.** Four input forces: (a) single impact; (b) double impacts; (c) multiple impacts; (d) random force.

### 3.2 Identification results

Multiple reference impact data were simulated through dynamic analyses of the FE model. White noise of 15% was added to the impacting forces and simulated structural accelerations to simulate observation noise, where 15% noise means that the standard deviation of the added noise is 15% of that of the simulated data. The procedure for processing the impact test data using the proposed method is described below for case 3. First, the system matrices  $A$ ,  $B$ ,  $C$ , and  $D$  are solved using the impact test data through Eqs. (7) and (8). As the model order  $n$  is unknown, the stabilization diagram shown in Figure 3 is used to determine the optimum  $n$  value by validating the stable poles of the structure with increasing model orders. The stabilization criteria state that the frequency error must be less than 20%, the damping error must be less than 100%, and the modal assurance criterion value must be greater than 0.98. From Figure 3, we can see that the structural poles become stable when  $n$  is greater than 20. Thus, we select  $n = 20$  considering the tradeoff between stable results and computation time. The frequencies and mode shapes of the studied structure that were identified in the first 4 modes are plotted in Figure 4, where the modal analysis results from the FE model are also provided for comparison.

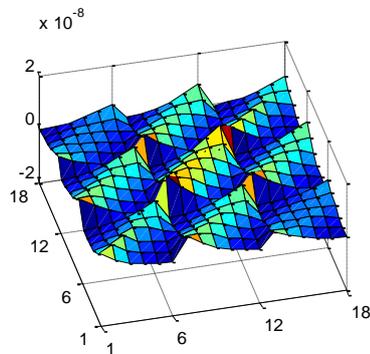


**Fig. 3.** Stabilization diagram.

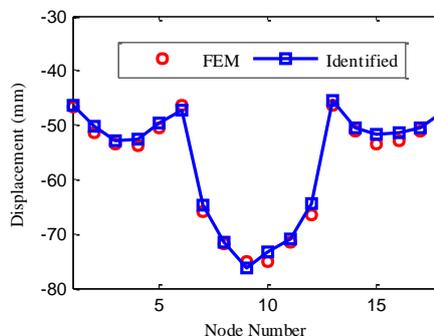


**Fig. 4.** Identified modal parameters.

After the basic modal parameters have been identified as described above, the modal scaling factor is calculated through Eq. (13). Figure 5 shows the structural flexibility matrix identified from Eq. (14). The structural displacements predicted under static loads of 1000 KN on nodes 1–18 are shown in Figure 6. It is apparent that the predicted deflections agree well with the exact deflections from the FE static analysis, which illustrates the effectiveness of the proposed method for accurate flexibility identification.



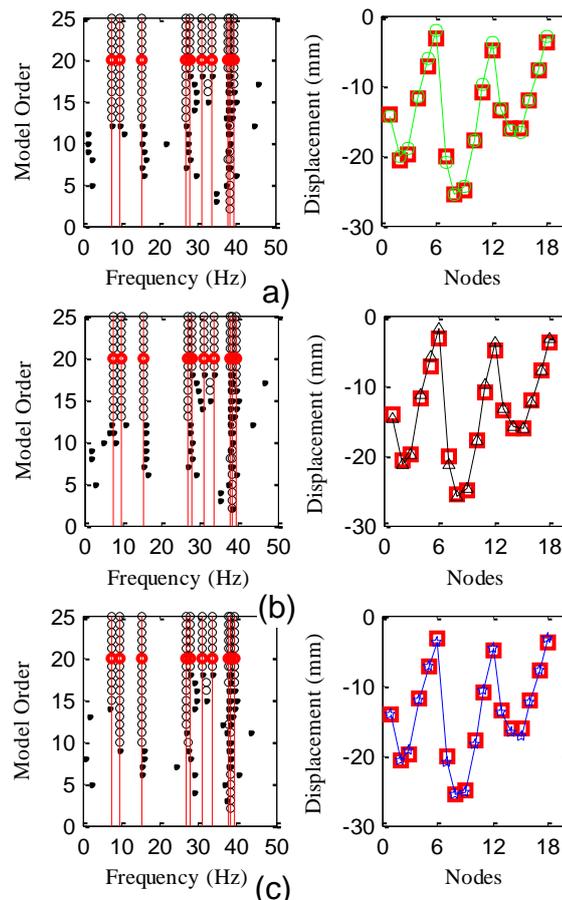
**Fig. 5.** Identified flexibility matrix.



**Fig. 6.** Deflection prediction from identified flexibility.

### 3.3 Effects of the input force type

To verify and illustrate the robustness of the proposed method to the input force type, multiple reference impact test data for cases 1, 2, and 4 using the different impacting forces shown in Figure 2 were processed by the proposed method. The stabilization diagrams and predicted displacements from the flexibility matrices identified in each case are plotted in Figure 7. The results given by the proposed method and the CMIF method are compared in Table 1. Clearly, the CMIF method only works well in the single impact case, whereas the proposed method is robust to changes in input force type.



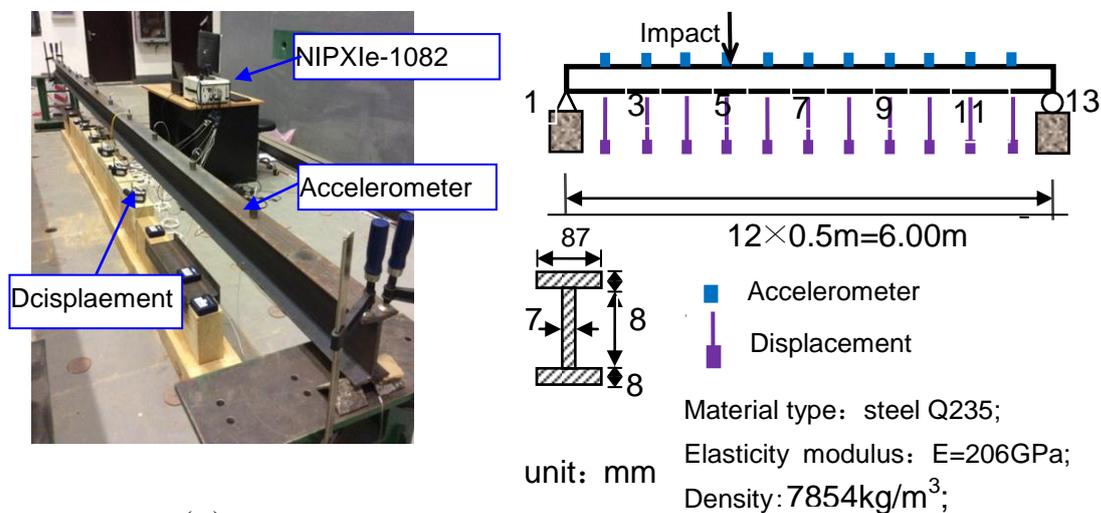
**Fig. 7.** Stabilization diagram and the corresponding displacement prediction: (a) single impact; (b) double impact; (c) random force.

**Table 1** Errors in the predicted deflections from the identified flexibility.

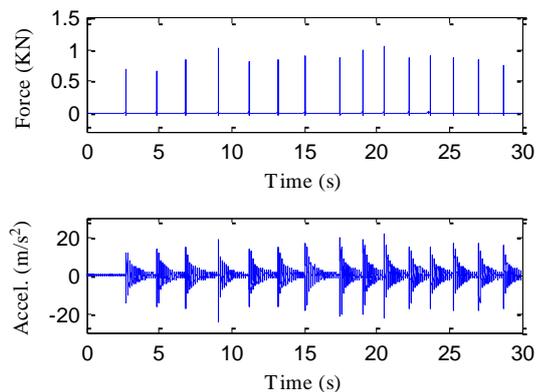
Excitation type	Proposed method	CMIF method
Case 1	2.67	2.80
Case 2	2.91	7.80
Case 3	1.74	11.24
Case 4	2.48	14.02

## 4 EXPERIMENTAL EXAMPLE

The proposed method was also applied to the flexibility identification of a simply supported beam in a laboratory by performing the multiple reference impact test, as shown in Figure 8. The beam had a length of 6 m with an I-shape section. Two ends of the structure were fixed in the vertical and longitudinal directions using F-type clips. Multiple reference impact tests and static tests were performed on the structure. In the impact test, hammer impacting was performed at nodes 4, 6, and 9 with random multiple clicks, and 11 PCB accelerometers were distributed at equal intervals across the entire beam to observe the structural responses. Typical time histories of the impacting forces and structural acceleration are shown in Figure 9. To verify the robustness of the proposed method to the force type, impacts with multiple clicks, which are not effectively handled by traditional methods, were performed in the test. The NI PXIe-1082 system was used for data acquisition, and the sampling frequency was set to 1000 Hz. A static test was performed to measure structural displacements under static loads. The results were then used to verify the accuracy of the predicted displacements and the identified flexibility. Eleven displacement transducers were distributed on nodes 2–12 to measure the structural deflections. Four static cases were tested: (1) 1.8 kN on node 7; (2) 1.8 kN on node 4; (3) 0.49 kN, 1.27 kN, and 0.49 kN on nodes 6, 7, and 8, respectively; and (4) 0.4 kN, 0.49 kN, 1.27 kN, 0.49 kN, 0.37 kN on nodes 5, 6, 7, 8, and 9, respectively.

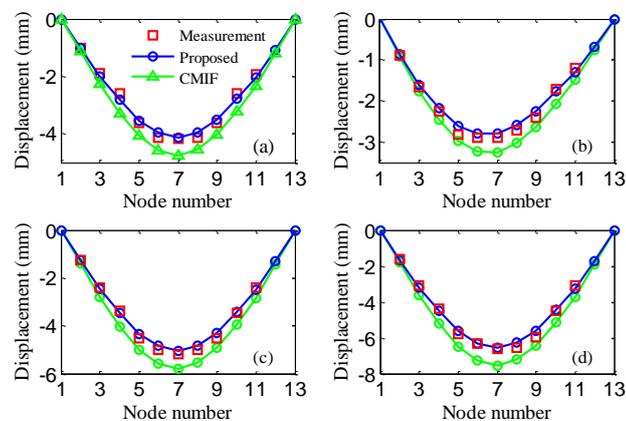


**Fig. 8.** Laboratory test of a simply supported beam: (a) photo; (b) sensor layout.



**Fig. 9.** Typical impact force and acceleration time histories.

The proposed method was applied to identify the structural flexibility in the time domain using the multiple reference impact test data. The flexibility of the simply supported beam was identified through Eq. (14). For comparison, the CMIF method was also used to process the same experimental data in the frequency domain. The predicted displacements from the proposed and the CMIF methods are compared with the measured displacements in Figure 10. It is clear that the values predicted by the proposed method are much closer to the measured values than those given by the CMIF method, which illustrates that the proposed method is robust to the input force type.



**Fig. 10.** Deflection prediction: (a) case 1; (b) case 2; (c) case 3; (d) case 4.

## 5 CONCLUSION

Based on the research reported in this paper, the following conclusions can be drawn:

- (1) Impact testing is able to estimate FRF magnitudes by measuring both input and output data. Thus, it has the potential to estimate the structural scaling factor and identify structural flexibility. In contrast, ambient vibrations cannot output FRF magnitudes, and can only identify basic modal parameters because the input is unknown.

- (2) Most subspace identification methods deal with ambient test data, and very few process impact test data. Previous approaches only identify basic modal parameters such as the frequency, damping ratio, and mode shapes. The subspace identification method proposed in this paper is the first to identify structural flexibility.
- (3) Numerical and experimental examples successfully illustrated that the proposed method is robust to different types of input force.

## ACKNOWLEDGMENTS

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