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On Pedestrian's Walking Load Identification

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ABSTRACT

Pedestrian's walking loads may induce severe vibrations to structures like footbridges, long-span floors and corridors leading to the so-called structural vibration serviceability issues. When designing structures subjected to pedestrian loads， reliable walking load model is crucial for accurate prediction of structural responses. However, it is not an easy task to measure the walking load directly. Inspired by the method for identifying moving load, the paper suggests an approach to identify pedestrian's walking load using structural responses. The method is based on modal superposition principle. With assumed modal shape functions for the line-like structure, analytical solution of the structural response due to pedestrian was derived and identification function was then established. In general, the identification matrix is ill-conditioned and the identification results are sensitive to measurement noise. To tackle this problem, the general regularization method is adopted. For the scenario of crowd moving load, it was simplified as a concentrated load or a moving load for identification purpose. Numerical examples show that accurate loads can be obtained by the suggested method for noise-free measurement. For noise-polluted case, the identification accuracy of walking load is acceptable for low noise level.

1. INTRODUCTION

Currently moving load identification is mainly focused on vehicle loading, for human-induced excitations acted on the footbridge or long-span floors have not been studied from the perspective of load identification. With the development of social economy, new structure form and high-strength lightweight materials have been widely used in public buildings, causing structural natural frequencies to be lower and lower. These structures are prone to vibrations under human-induced excitation, so the structural response under pedestrian's walking loads can't be ignored, especially for footbridges in urban or tourism area. Accurate estimate of pedestrian's walking load is very important for the structural design, control and vibration serviceability assessment. However, it is not an easy task to measure the walking load directly. Inspired by the method for identifying moving load, this paper suggests an approach to identify pedestrian's walking load using structural responses.

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In this paper, an attempt is made for pedestrian's walking load identification, on the basis of modal superposition principle, and assuming modal shape functions for the line-like structure (Law 1997), analytical solution of the structural response due to pedestrian was derived and identification function was then established. Numerical examples are given to show the application of the proposed method.

2. THEORY OF WALKING LOAD IDENTIFICATION

Suppose one person walking on a simply supported beam at a constant speed, as shown in Fig.1. The beam is assumed to be of constant cross-section with constant mass per unit length, having linear, viscous proportional damping with small deflections, and the effects of shear deformation and rotary inertia are not considered.

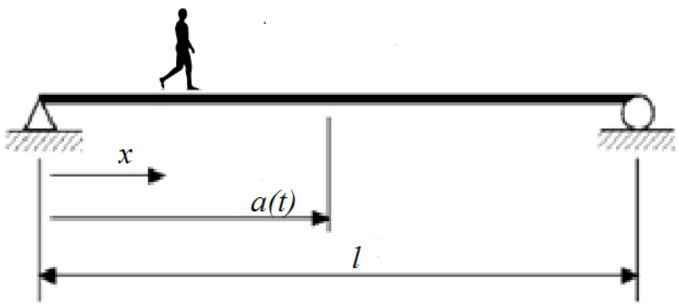

Fig.1 Simply supported beam subjected to pedestrian's walking load

The equation of motion can be written as:

$$\rho \frac{\partial^2 u(x,t)}{\partial t^2} + C \frac{\partial u(x,t)}{\partial t} + EI \frac{\partial^4 u(x,t)}{\partial x^4} = \delta(x - a(t)) f(t) \tag{1}$$

where $u(x,t)$ is the beam deflection at point x and time t, ρ is the mass per unit length, C is the viscous damping parameter, E is the Young's modulus of the material, I is the second moment of inertia of the beam cross-section, l is the length of the beam, $a(t)$ is the time varying load point, $f(t)$ is the time varying walking load, c is the walking speed, and $\delta(t)$ is the Dirac delta function.

Based on modal superposition, the dynamic deflection $u(x,t)$ can be described as follows:

$$u(x,t) = \sum_{n=1}^{\infty} \Phi_n(x) q_n(t) \tag{2}$$

where n is the mode number, $\Phi_n(x)$ is the mode shape function of the nth mode, and the $q_n(t)$ are the nth mode amplitudes. Substituting Eq. (2) into Eq. (1), we obtain

$$\frac{d^2 q_n(t)}{dt^2} + 2\xi_n w_n \frac{dq_n(t)}{dt} + w_n^2 q_n(t) = \frac{1}{M_n} p_n(t) \tag{3}$$

where w_n is the modal frequency of the nth mode, ξ_n is the damping ratio of the nth mode, and M_n is the modal mass of the nth mode, and $p_n(t)$ is the modal fore. Based on assumptions for the beam, the modal parameters of the beam can be calculated as follows:

$$w_n = (n\pi / l)^2 \sqrt{EI / \rho}, \qquad \Phi_n(x) = \sin(n\pi x / l) \tag{4}$$

$$M_n = \rho l / 2, \qquad p_n(t) = f(t)\sin(n\pi a(t) / l) \tag{5}$$

For real structures, the modal parameters can be obtained from the finite element model or modal testing.

Eq. (3) can be solved in the time domain:

$$q_n(t) = \frac{1}{M_n}\int_0^t h_n(t-\tau)\, p_n(\tau)\, d\tau \tag{6}$$

where $$h_n(t) = (1/w_n^{'})e^{-\xi_n w_n t}\sin(w_n^{'} t), \quad w_n^{'} = w_n\sqrt{1-\xi^2} \tag{7}$$

Substituting Eq. (4) and Eq. (6) into Eq. (2), the dynamic deflection of the beam at point x and time t can be obtained (Chan 1999).

$$u(x,t) = \sum_{n=1}^{\infty}\frac{2}{\rho l w_n^{'}}\sin(\frac{n\pi x}{l})\int_0^t e^{-\xi_n w_n (t-\tau)}\sin w_n^{'}(t-\tau)\sin\frac{n\pi a(t)}{l} f(\tau)\, d\tau \tag{8}$$

Eq. (8) can be rewritten in discrete terms as

$$u(i) = \frac{1}{M_n w_n^{'}}\sum_{n=1}^{\infty}\Phi_n(x)\sum_{j=0}^{i} e^{-\xi_n w_n \Delta t(i-j)}\sin(w_n^{'}\Delta t(i-j))\sin(\frac{n\pi a(j)}{l}) f(j)\Delta t \tag{9}$$

$$i = 0,1,2\cdots N_t$$

where Δt is the sampling interval and N_t+1 is the number of sample points.

The response of mode n is

$$u(i)_n = \frac{1}{M_n w_n^{'}}\Phi_n(x)\sum_{j=0}^{i} e^{-\xi_n w_n \Delta t(i-j)}\sin(w_n^{'}\Delta t(i-j))\sin(\frac{n\pi a(j)}{l}) f(j)\Delta t \tag{10}$$

Let

$$D_{xn} = \frac{2}{\rho l w_n^{'}}\sin(\frac{n\pi x}{l}) \tag{11}$$

$$H_n(k) = e^{-\xi_n w_n k\Delta t}\sin(w_n^{'} k\Delta t)\Delta t \tag{12}$$

$$S_n(k) = \sin(\frac{n\pi a(k)}{l}) \tag{13}$$

Arranging Eq. (10) into matrix form,

$$\begin{Bmatrix} u(0) \\ u(1) \\ u(2) \\ u(3) \\ \vdots \\ u(N_t) \end{Bmatrix}_n = \begin{Bmatrix} H_n(0)S_n(0) & 0 & 0 & \cdots & 0 \\ H_n(1)S_n(0) & H_n(0)S_n(1) & 0 & \cdots & 0 \\ H_n(2)S_n(0) & H_n(1)S_n(1) & H_n(0)S_n(2) & \cdots & 0 \\ H_n(3)S_n(0) & H_n(2)S_n(1) & H_n(1)S_n(2) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ H_n(N_t)S_n(0) & H_n(N_t-1)S_n(1) & H_n(N_t-2)S_n(2) & \cdots & H_n(N_t-N_B)S_n(N_B) \end{Bmatrix} \begin{Bmatrix} f(0) \\ f(1) \\ f(2) \\ f(3) \\ \vdots \\ f(N_B) \end{Bmatrix} \quad (14)$$

Assuming the initial condition as

$$f(0)=0, \quad f(N_t)=0, \quad u(0)=0, \quad u(1)=0 \quad (15)$$

Eq. (14) can be condensed as

$$\begin{Bmatrix} u(2) \\ u(3) \\ u(4) \\ u(5) \\ \vdots \\ u(N_t) \end{Bmatrix}_n = D_{xn} \begin{Bmatrix} H_n(1)S_n(1) & 0 & 0 & \cdots & 0 \\ H_n(2)S_n(1) & H_n(1)S_n(2) & 0 & \cdots & 0 \\ H_n(3)S_n(1) & H_n(2)S_n(2) & H_n(1)S_n(3) & \cdots & 0 \\ H_n(4)S_n(1) & H_n(3)S_n(2) & H_n(2)S_n(3) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ H_n(N_t-1)S_n(1) & H_n(N_t-2)S_n(2) & H_n(N_t-3)S_n(3) & \cdots & H_n(N_t-N_B+1)S_n(N_B-1) \end{Bmatrix} \begin{Bmatrix} f(1) \\ f(2) \\ f(3) \\ f(4) \\ \vdots \\ f(N_B-1) \end{Bmatrix} \quad (16)$$

Eq. (16) is simply rewritten as

$$\underset{(N_t-1)\times 1}{u} = \underset{(N_t-1)\times(N_B-1)}{B} \; \underset{(N_B-1)\times 1}{f} \quad (17)$$

3. REGULARIZATION

The natural frequencies and mode shapes obtained from modal testing and modal analysis are subject to measurement error. Noise contamination in the test data has adverse effect on the accuracy of the identified walking load. In general, the walking load can be identified from Eq. (16) by the least-squares method, but the result would be unbound. A regularization technique can be used to solve the ill-conditioned problem in the form of minimizing the function parameter (Y. Xiao 2013). The generalized cross-validation and L-curve method are employed to determine the optimal regularization parameter (Zhu 2002).

$$J(P,\lambda) = \left\| B*P-\varepsilon \right\|^2 + \lambda \left\| P \right\|^2 \quad (18)$$

where λ is the non-negative regularization parameter

4. SIMULATION AND RESULTS

To check the correctness and effectiveness of this method, there is a simply supported beam subject to walking load. The parameters of the beam are as followed: $EI=1\times10^9(\mathrm{N}*m^2)$, $\rho=600kg/m$, $l=20\mathrm{m}$, $\xi=0.02$, $N=3$, human walking speed $v=1.5m/s$, step length $\Delta l=0.75\mathrm{m}$, and suppose that the first step is 0.5m from the left support. $N_t=N_B=1300$, and the responses of $0.25*l$, $0.45*l$ and $0.8*l$ are used in the identification. (G. Ding 2016) studied the single step walking load, so this paper assumes that each step load is the same and adopts one of the cases.

$$f1=600(0.81+0.1632\sin(\frac{2\pi}{T}t-\frac{\pi}{2})+0.3442\sin(\frac{4\pi}{T}t-\frac{\pi}{2})+0.1470\sin(\frac{6\pi}{T}t-\frac{\pi}{2}) \quad (19)$$

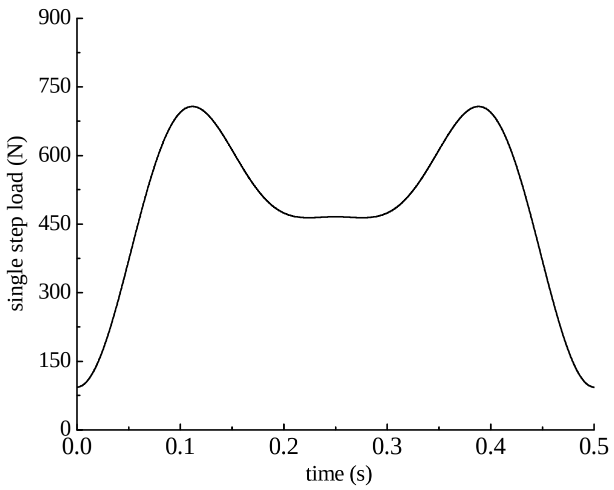

Fig.2 Single step load of walking

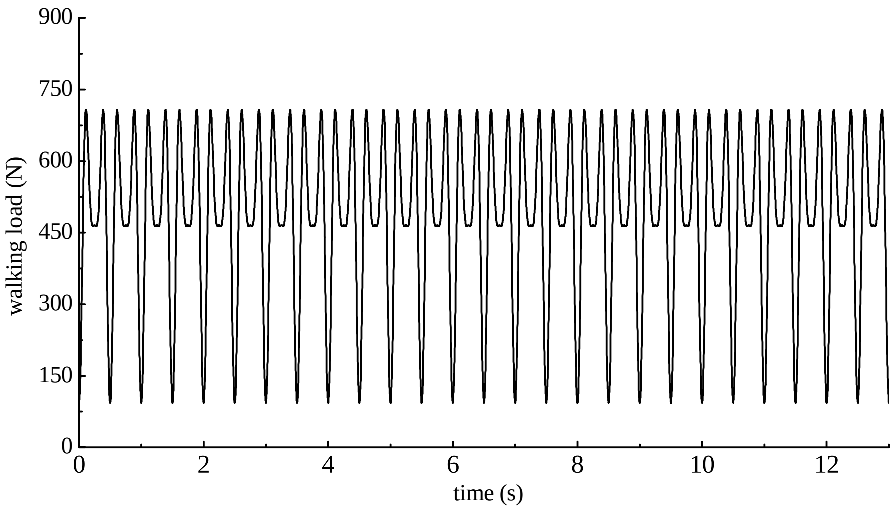

Fig.3 Simulated time varying walking load *f(t)*

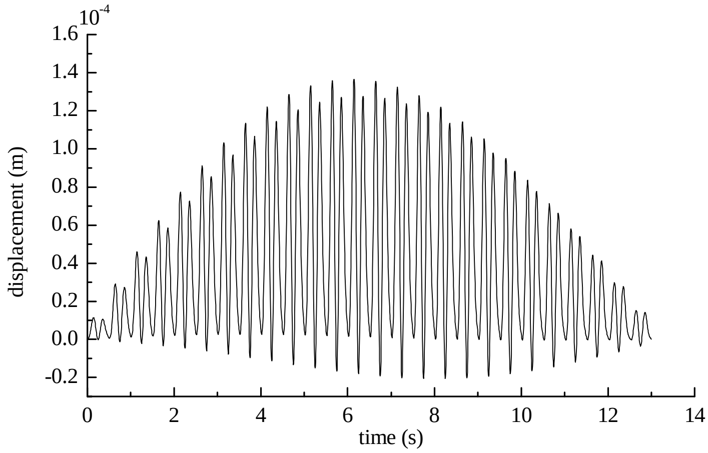

Fig.4 The displacement of 0.25**l* (E_p=0)

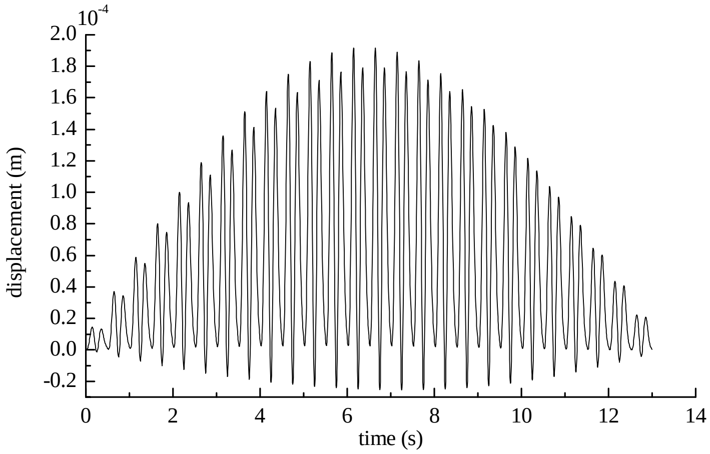

Fig. 5 The displacement of 0.45**l* (E_p=0)

Besides, white noise is added to the calculated responses to simulate the polluted measurements.

$$u = u_{calculated} + E_p \times \left\| u_{calculated} \right\| \times N_{oise} \tag{20}$$

where E_p is a specified error level, N_{oise} is a standard normal distribution vector. The estimation error can be calculated by the followed equation:

$$Error = \frac{\left\| f_{identified} - f_{true} \right\|}{\left\| f_{true} \right\|} \times 100\% \tag{21}$$

Eq. (18) and Eq.(19) are used to identify the walking load and the following results are obtained for different cases.

(1) If E_p=0, it means no noises are added to the measured displacements (Fig.4 and Fig. 5), accurate results are obtained. This confirms that using displacements to identify the walking load is feasible.

(2) If E_p=1%, the simulation of the displacements are showed in the Fig.6 and Fig.7, and the walking load can be identified as Fig.8 , *Error*=10.6%.

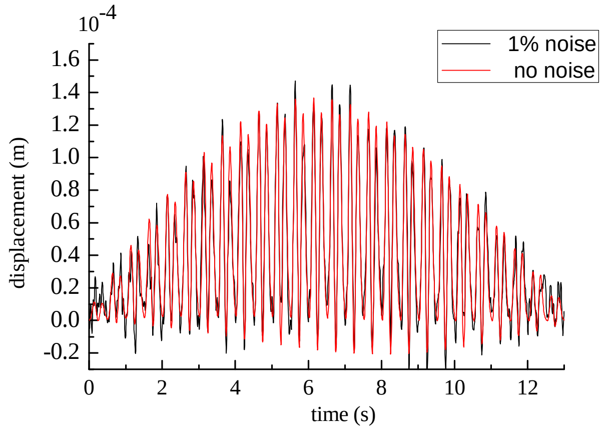

Fig.6　The displacement of 0.25*l with 1% noise

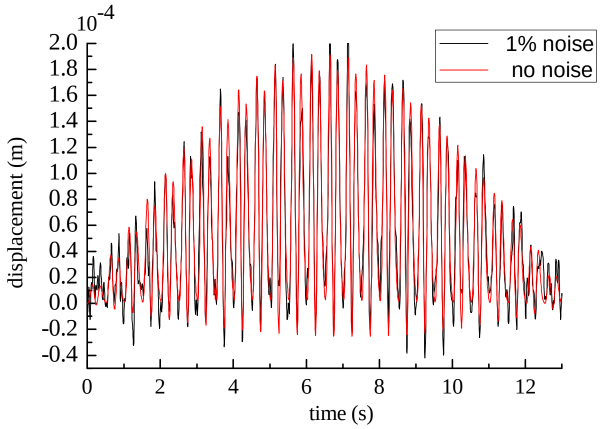

Fig.7　The displacement of 0.45*l with 1% noise

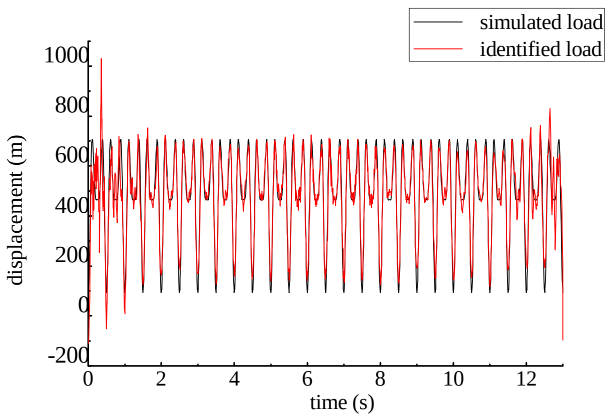

Fig.8　The identified load form polluted displacement (E_p=0.01)

5. CONCLUSIONS

From theoretical studies and computation simulation, some conclusions can be obtained. First it is feasible to use measured responses such as displacement to identify pedestrian's walking load in the time domain. Although the identification matrix is ill-conditioned, the general regularization method is a good solution to this problem. When measured responses are polluted, the identified result is satisfied by preprocessing the data and least square method.

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