$$\Xi_c = \frac{\mathbf{\Phi}_i^{\mathrm{T}}\mathbf{C}\mathbf{\Psi}_1}{M_i}\dot{p}_1 + \ldots + \frac{\mathbf{\Phi}_i^{\mathrm{T}}\mathbf{C}\mathbf{\Psi}_j}{M_i}\dot{p}_j + \ldots + \frac{\mathbf{\Phi}_i^{\mathrm{T}}\mathbf{C}\mathbf{\Psi}_n}{M_i}\dot{p}_n \tag{10b}$$

$$\Xi_k = \frac{\mathbf{\Phi}_i^{\mathrm{T}}\hat{\mathbf{K}}\mathbf{\Psi}_1}{M_i}p_1 + \ldots + \frac{\mathbf{\Phi}_i^{\mathrm{T}}\hat{\mathbf{K}}\mathbf{\Psi}_j}{M_i}p_j + \ldots + \frac{\mathbf{\Phi}_i^{\mathrm{T}}\hat{\mathbf{K}}\mathbf{\Psi}_n}{M_i}p_n \tag{10c}$$

The coupling phenomenon of co-ordinates q_i and p_i is hidden in Eqs.(10). Neglecting the coupling co-ordinates in Eq.(10) results in the following,

$$\Xi_m \approx \frac{\mathbf{\Phi}_i^{\mathrm{T}}\mathbf{M}\mathbf{\Psi}_i}{M_i}\ddot{p}_i = \ddot{p}_i \tag{11a}$$

$$\Xi_c \approx \frac{\mathbf{\Phi}_i^{\mathrm{T}}\mathbf{C}\mathbf{\Psi}_i}{M_i}\dot{p}_i = 2\zeta_i\omega_i\dot{p}_i \tag{11b}$$

$$\Xi_k \approx \frac{\mathbf{\Phi}_i^{\mathrm{T}}\hat{\mathbf{K}}\mathbf{\Psi}_i}{M_i}p_i = \hat{\omega}_i^2 p_i \tag{11c}$$

According to the expression Eq.(1), neglecting the coupling co-ordinates gives the controlling equation of ASBS as,

$$\ddot{\tilde{q}}_i + 2\zeta_i\omega_i\dot{\tilde{q}}_i + f_i\left(r_{iy}, \lambda_{id}, \tilde{q}_i\right) = -\Gamma_i\ddot{x}_g \tag{12}$$

where the new co-ordinate $\tilde{q}_i$ for the bilinear elastic system is equal to p_i+q_i; the factors of r_{1y}, λ_{1d} like the parameters R_{1y}, α_1 defined in Eq.(2) and Eq.(3) which are the “yielding” strength reduction factor, frequency degradation factor for the ASBS, respectively, and can be expressed by,

$$r_{iy} = \frac{q_{il}}{q_{iy}} = \frac{f_{il}}{f_{iy}} \tag{13}$$

$$\lambda_{id} = \frac{\hat{\omega}_i^2}{\omega_i^2} \tag{14}$$

where q_{il} (or f_{il}) is the maximal co-ordinate value (or restoring force) of the linear elastic system; q_{iy} (or f_{iy}) is the “yielding” co-ordinate value (or restoring force), which has the following relation with x_{1y} in Eq.(2),

$$\phi_{1i}q_{iy} = x_{1y} \tag{15}$$

where ϕ_{1i} is the 1st element of $\mathbf{\Phi}_i$. So the restoring force f_i in Eq. (2) can be expressed by,

$$f_i\left(r_{iy}, \lambda_{id}, \tilde{q}_i\right) = \begin{cases} \omega_i^2 \tilde{q}_i, & if\ \tilde{q}_i \le q_{iy} \\ \omega_i^2 q_{iy} + \hat{\omega}_i^2\left(\tilde{q}_i - q_{iy}\right), & if\ \tilde{q}_i > q_{iy} \end{cases} \tag{16}$$

4. NUMERICAL VALIDATION

In order to illustrate the accuracy for the approximations, Eqs.(11), an eight-story structure example is employed (Li et al., 2015). The damping ratio for this structure is assumed to be 5%. This paper considers that the additional negative stiffness property only exists in the 1st story, and other stories maintain the linear elastic property.

The MNS1 response can be estimated by the ASBS in Eq.(12). Due to the approximate equations Eqs.(11), the ASBS would lead to some errors in the response of the MNS1. This section will analyze the accuracy. The 1st floor displacement and base shear are considered, which can be estimated by,

$$\tilde{x}_{ji} = \begin{cases} \tilde{q}_i \Phi_{ji}, & if\ \tilde{q}_i \le q_{iy} \\ \left(\tilde{q}_i - q_{iy}\right)\Psi_{ji} + q_{iy}\Phi_{ji}, & if\ \tilde{q}_i > q_{iy} \end{cases} \tag{17}$$

$$\tilde{V}_i = \begin{cases} \mathbf{E}^{\mathrm{T}}\mathbf{K}\mathbf{\Phi}_i \tilde{q}_i, & if\ \tilde{q}_i \le q_{iy} \\ \mathbf{E}^{\mathrm{T}}\hat{\mathbf{K}}\mathbf{\Psi}_i\left(\tilde{q}_i - q_{iy}\right) + \mathbf{E}^{\mathrm{T}}\mathbf{K}\mathbf{\Phi}_i q_{iy}, & if\ \tilde{q}_i > q_{iy} \end{cases} \tag{18}$$

where *i* represents the *i*-th mode; *j* is the *j*-th floor; $\tilde{x}_{ji}$ and $\tilde{V}_i$ are the estimated *j*-th floor displacement and base shear under *i*-th mode, respectively.

The combined displacement and base shear for the first three modes are compared with the real response in Fig. 2.

(a) (b)

Fig. 2 Comparisons of real and estimated displacement on 1st floor and base shear

In Fig. 2, the errors of peak values for the 1st floor displacement and the base shear are 5.43% and 0.92%, respectively. Errors from the evaluation by Eq.(12) are due to the approximation Eqs.(11), which means that the approximation in Eq. (11) is reasonable.

5. CONCLUSIONS

This paper develops a new method using the single degree of freedom bilinear elastic system (ASBS) to approximately estimate the response of the multi degree of freedom (MDOF) structure with the negative stiffness (NS) in the 1st story (MNS1) based on mode superposition method, which neglects the coupling phenomenon in the co-ordinates.

An eight-story numerical example is employed to illustrate the accuracy of approximation of the proposed method. The results shows that the approximate procedure provides good estimates of floor displacements and base shears.

REFERENCES

Chopra, A. (2001), "Dynamics of Structures: Theory and Applications to Earthquake Engineering", Englewood Cliffs, New Jersey, U.S.A., Prentice-Hall.

Kalogeropoulos, G. I. & Tsonos, A. G. (2014), "Effectiveness of R/C jacketing of substandard R/C columns with short lap splices," *Structural Monitoring and Maintenance*,**1**, 273-292.

Li, G., Li, H. & Zhang, Y. (2015), "Displacement estimation of nonlinear structures using the force analogy method," *The Structural Design of Tall and Special Buildings,* **24,** 59–72.

Nagarajaiah, S., Reinhorn, A. M., Constantinou, M. C., Taylor, D., Pasala, D. T. R. & Sarlis, A. A. (2010), "True adaptive negative stiffness: A new structural modification approach for seismic protection," *Proceedings of the 5th World Conf. on Structural Control and Monitoring.* Univ. of Tokyo, Japan, International Association of Structural Control and Monitoring.

Pasala, D. T. R., Sarlis, A. A., Nagarajaiah, S., Reinhorn, A. M., Constantinou, M. C. & Taylor, D. (2013), "Adaptive negative stiffness: A new structural modification approach for seismic protection," *Journal of Structural Engineering, ASCE,* **139,** 1112–1123.

Reinhorn, A. M., Viti, S. & Cimellaro, G. P. (2005), "Retrofit of structures: Strength reduction with damping enhancement," *Proceedings of the 37th UJNR Panel Meeting on Wind and Seismic Effects.* Tsukuba-shi, Japan, Public Works Research Institute

Tsonos, A. G. (2014), "An innovative solution for strengthening of old R/C structures and for improving the FRP strengthening method," *Structural Monitoring and Maintenance*, **1**, 323-338.

Viti, S., Cimellaro, G. P. & Reinhorn, A. M. (2006), "Retrofit of a hospital through strength reduction and enhanced damping," *Smart Structures and Systems,* **2**, 339-355.