The 2016 World Congress on
The 2016 Structures Congress (Structures16)
Jeju Island, Korea, August 28-September 1, 2016

Instantaneous frequency identification based on synchrosqueezing wavelet transform

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1. Introduction

There is now a consensus that nonlinearity is generic in the area of civil engineering, and linear behavior is an exception. Nonlinear behavior of structures often produce non-stationary signals, which brings great challenges to traditional signal processing methods such as FFT and so on. To approach this dearth of investigating local features of non-stationary signals, time-frequency representation (TFR) analysis has recently received increasing acceptance in the field of nonlinear parameter identification. So far, a variety of methods for non-stationary signal processing have been proposed in literatures, which mainly include short-time Fourier transform(STFT), Gabor transform ,Cohen class quadratic distribution, Hilbert-Huang transform(HHT) (Huang 1999a, b) and continues wavelet transform (CWT) (Kijewski 1999, 2007). Among these methods mentioned above, HHT and CWT play the most important role without adding more complexity. In 1998, Huang, et al explored a sifting procedure called empirical modal decomposition (EMD) to decompose multi-component signal to a finite number of intrinsic mode functions (IMFs),then Hilbert spectral analysis was employed to realize the extraction of instantaneous characteristics of non-stationary signals and the identification of time-varying parameters. The latest advances of HHT is ensemble empirical modal decomposition (EEMD) (Zhao 2003). This new noise-assisted dada analysis method consists of sifting an ensemble of white noise-added signal and treats the mean as final true result. However, HHT belongs to empirical local analysis method and encounter a few problems in practical application. For example, HHT still have

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some difficulties in negative frequency, distortion, non-convergence EMD and the ambiguity of IMF definition (Huang 2005). In addition, HHT can't distinguish closely-spaced modal responses, especially signals with overlapping modal frequency(Chen 2012). For response of this kind of signal generally tends to exist in several adjacent decomposited signals, thus it needs further reassignment. Compared HHT with CWT, Yan et al. (2006)demonstrated that wavelet transform is superior to HHT for identification of closely-spaced mode.

As a new linear TFR method, wavelet transform is capable of adjusting time windows and frequency window adaptively to facilitate multiresolution analysis. As wavelet transform has remarkable advantages, researchers have recently done a lot on parameter identification by using it. Ruzzene et al. (1997) introduced complex wavelet transform to estimate natural frequency and viscous damping ratios of free response. The accuracy of this method is confirmed by applying it to a numerical example and the acceleration response from a real bridge under ambient excitation. Afterwards, Hou et al. (Hou 2006) employed a wavelet-based structural health monitoring technique to extract instantaneous mode shape information from responses during an earthquake event; furthermore, a confidence index (CI) is presented to validate the results obtained. On this basis, Xu and Shi (2012) combined wavelet theory and state-space method (wavelet-based state-space method) for the identification of dynamic parameters in linear time-varying system, and numerical results verify its robustness as well as effectiveness. In virtue of end effect and choice of time-frequency localization highlighted in wavelet transform, Le et al studied three complex-valued mother wavelets, and put forward correct choice of mother wavelets parameters to estimate modal parameters accurately.

It is true that wavelet transform has been widely used in time-varying and nonlinear parameter identification, but Kijewski and Kareem (2003) points out that wavelet transform can't always provide finer enough frequency resolution for long period signal components which is marked for response of civil engineering structures. Concrete manifestation of this problem is low-frequency signal component distributes widely and looks blurry in the time-frequency plane. Therefore how to get a clear view about time-frequency curves of measured signal remain a problem not well resolved. Recently Daubechies (2011) explored a completely new method called synchrosqueezing wavelet transform, which is a combination of wavelet analysis and reallocation method. This procedure can effectively reassign time frequency curves to get more accurate results, and decompose an arbitrary signal to linear superposition of several approximate harmonics at ease. Furthermore, synchrosqueezing wavelet transform can be used to extract instantaneous frequency (IF) even if the waveforms are non-harmonics. Wu (2011) presented an EMD-inspired synchrosqueezing method to study a signal with close IFs, and the results showed a huge success. Li (2012, 2014) proposes to use a generalized synchrosqueezing transform (GST) approach to detect and diagnose gearbox faults due to the diffusions of the TFR energy along time or frequency axes.

In conclusion, synchrosqueezing wavelet transform can capture the flavor and philosophy of the EMD approach and give insight into the structure of component signals. Furthermore, it allows individual reconstruction of these components. Such advantages are undoubtedly useful, and some related theoretical researches have been done on it, however, practical applications are still relatively rare. It is important to note that most systems in civil engineering belong to nonlinear systems and generated non-stationary responses often possess long period or low frequency components, so parameter identification of these systems remains a longstanding challenge to us. In this paper, synchrosqueezing wavelet transform is first introduced to the field of civil engineering to estimate to IFs of nonlinear systems. Examples of a Duffing nonlinear system with free vibration and two-story shear building with forced vibration validate the accuracy and effectiveness of this method. Then different wavelet mother functions are provided to study the influence wavelet transform applied to synchrosqueezing algorithm, the results show that synchrosqueezing wavelet transform is less affected by the selection of wavelet mother functions, and the identified instantaneous frequency has strong stability and robustness. Besides numerical simulation, a cable structure test is set up with linearly and sinusoidally varying tension forces, respectively, which results in a time-varying and nonlinear stiffness. Synchrosqueezing wavelet transform is next used to estimate IFs and the identified results are well in agreement with calculation results in theoretical calculation.

2. Synchrosqueezing wavelet transform

As a powerful tool for analyzing signals with non-stationary behavior, TFR is capable of recognizing instantaneous frequency and instantaneous amplitude at every time point, so it inevitable that time-frequency analysis gradually become an area of intense investigation in modern signal processing field. Practical measured time-varying signal usually includes several components, and each component has its own local features. A typical time varying signal can be expressed as the sum of N intrinsic mode functions and a residual.

$$x(t) = \sum_{i=1}^{N} x_i(t) + r(t) \tag{1}$$

in which, each IMF $x_i(t) = A_i(t)\cos\left(\phi_i(t)\right)$ is an oscillating function. Generally, the change in time of $A_i(t)$ and $\phi'_i(t)$ is much slower than the change of $\phi_i(t)$ itself, and $r(t)$ represents noise or observation error. Signals of form (1) arise naturally in the area of civil engineering. All we need to do is extracting the amplitude factor $A_i(t)$ and IF

$\phi'_i(t)$ for each *i* by refining time frequency curves. As a special reallocation method, synchrosqueezing aims to refine wavelet transform coefficient $W(t,\omega)$ by assigning its value to different point (t',ω') in the time-frequency plane according to the local behavior of $W(t,\omega)$ around (t,ω).

As the name implies, synchrosqueezing wavelet transform is based on wavelet transform, so it is essential to introduce wavelet transform first. For a given mother wavelet function, the CWT of signal *x*(*t*) is defined by

$$W_x(a,b) = \int_{-\infty}^{\infty} x(t)\frac{1}{\sqrt{a}}\overline{\psi\left(\frac{t-b}{a}\right)}\,dt \tag{2}$$

Where *a* and *b* is scale factor and dilation factor respectively, and $\overline{\psi\left(\frac{t-b}{a}\right)}$ represents the complex conjugate of $\psi\left(\frac{t-b}{a}\right)$. The mapping between scale factor *a* and signal frequency ω facilitate displaying wavelet coefficients in time-frequency plane, so several different algorithms can be employed to support the extraction of wavelet ridge and the identification of IFs. Nevertheless, a meaningful research (Daubechies 1996) indicates that wavelet coefficients itself is an oscillating function of time even for the simplest harmonic wave function. If the mother wave function ψ has fast decay, its Fourier transform $\hat{\psi}(\xi)$ is approximately equal to zero in the negative frequencies: $\hat{\psi}(\xi)=0$, for $\xi<0$, and is concentrated around $a=\omega_0/\omega$. Take $x(t)=A\cos(\omega t)$ for example, we can rewrite $W_x(a,b)$ by Plancherel's theorem, as

$$W_x(a,b) = \frac{1}{2\pi}\int \hat{x}(\xi)\sqrt{a}\overline{\hat{\psi}(a\xi)}e^{ib\xi}d\xi = \frac{A}{4\pi}\int[\delta(\xi-\omega)+\delta(\xi+\omega)]\sqrt{a}\overline{\hat{\psi}(a\xi)}e^{ib\xi}d\xi$$
$$= \frac{A}{4\pi}\sqrt{a}\overline{\hat{\psi}(a\omega)}e^{ib\omega} \tag{3}$$

Here, $\hat{x}(\xi)$ is the Fourier transform of signal *x*(*t*), while $\hat{\psi}(\xi) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\psi(t)\,e^{-i\xi t}dt$ represents ψ in frequency domain. For the purpose of accurately computing the IFs of signals, a research (Daubechies 1996) indicates that although $W_x(a,b)$ is spread out in *a*, its oscillatory behavior in *b* points to the original frequency ω, no matter what the value of *a* would be. Consequently, IF is suggested to be preliminarily estimated by taking derivatives of wavelet coefficients. The formula of computation is shown as follows:

$$\omega_x(a,b) = \begin{cases} \frac{-i\partial_b W_x(a,b)}{W_x(a,b)} & |W_x(a,b)|>0 \\ \infty & |W_x(a,b)|=0 \end{cases} \tag{4}$$

Obviously, we can use Eq. (4) to build a map between (a,b) and $(\omega_x(a,b),)$ without any difficulty. In the next synchrosqueezing step, the energy from time-scale plane is transferred to the time-frequency plane, according to the map built by Eq. (4).

The frequency variable ω and scale factor *a* were "binned", namely discretized, i.e. $W_x(a,b)$ was computed only at discrete points a_i, with $a_i - a_{i-1} = (\Delta a)_i$, and its synchrosqueezing value was likewise determined merely at the centers ω_l of closed intervals $[\omega_l - \frac{1}{2}\Delta\omega, \omega_l + \frac{1}{2}\Delta\omega]$, with $\omega_l - \omega_{l-1} = \Delta\omega$. By summing these different contributions, synchrosqueezing wavelet transform of *x*(*t*) is obtained.

$$T_x(\omega_l, b) = \sum_{a_i:|\omega_x(a,b)-\omega_l|\leq\Delta\omega/2} W_x(a,b)a_i^{-3/2}\,(\Delta a)_i \tag{5}$$

If frequency ω and scale *a* are treated as continuous variables, the analog of the above Eq. is

$$T_x(\omega, b) = \int_{A(b)} \mathrm{W_x}(a,b)a^{-3/2}\delta(\omega(a,b) - \omega)\,\mathrm{d}a \tag{6}$$

On the surface, synchrosqueezing is similar to TFR methods, however, there are some differences between synchrosqueezing and standard TFR techniques such as STFT, CWT and Wiger-Vill distribution (Thakur 2013). Synchrosqueezing wavelet transform can extract and delineate components by sharpening time-frequency spectrum, and unlike most TFR methods, it allow individual reconstruction of these components. In other words, synchrosqueezing wavelet transform is invertible, so original signal $x(b)$ can be reconstructed by performing inverse transform to $T_x(\omega_l, b)$.

The following argument indicates that the original signal can still be reconstructed after synchrosqueezing wavelet transform is performed. We have

$$\int_0^\infty W_x(a,b)a^{-3/2}\ da = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_0^\infty \hat{x}(\xi)\overline{\hat{\psi}(a\xi)}e^{ib\xi}a^{-1}dad\xi = \frac{1}{2\pi}\int_0^{\infty}\int_0^\infty \hat{x}(\xi)\overline{\hat{\psi}(a\xi)}e^{ib\xi}a^{-1}dad\xi$$

$$= \int_0^\infty \overline{\hat{\psi}(\zeta)}\frac{\mathrm{d}\zeta}{\zeta}\cdot\frac{1}{2\pi}\int_0^\infty \hat{\mathrm{x}}(\xi)\mathrm{e}^{\mathrm{i}b\xi}\mathrm{d}\xi \tag{7}$$

By defining a normalizing constant $C_\psi = \frac{1}{2}\int_0^\infty \overline{\hat{\psi}(\zeta)}\frac{d\zeta}{\zeta}$, the original signal can be estimated as

$$x(b) = \Re\mathrm{e}\left[C_\psi^{-1}\left(\int_0^\infty W_x(a,b)a^{-3/2}da\right)\right] \tag{8}$$

In the piecewise constant approximation corresponding to the binning in a, Eq. (8) becomes

$$x(b) \approx \Re\mathrm{e}\left[C_\psi^{-1}\sum_i W_x(a,b)\,a_i^{-3/2}(\Delta a)_i\right] = \Re\mathrm{e}\left[C_\psi^{-1}\sum_l T_x(\omega_l,b)(\Delta\omega)\right] \tag{9}$$

In brief, for a wide range of asymptotic signal according with the synchrosqueezing assumptions, each component of analyzed signal is well concentrated in the time-frequency plane and thus can be estimated successfully, provided a sufficiently fine division of frequency bins $\{\omega_l\}$. More of the synchrosqueezing algorithm (definitions, estimates and proofs) is detailed in the literature (Daubechies 2011).

3. Numerical simulations

3.1 IF extraction of Duffing system with free vibration

Classical Duffing Eq. is usually employed to simulate nonlinear motion of mass-spring-damper systems. A Duffing Eq. in this example is given by Feldman (2011).

$$\ddot{x} + 0.05\dot{x} + x + 0.01x^3 = 0 \tag{10}$$

The motion begins with $x_0 = 10$, $\dot{x} = 0$, and its response can be simulated using 4th Runge-Kutta method with time interval of 0.1 seconds. Fig. 1 shows the results of displacement response.

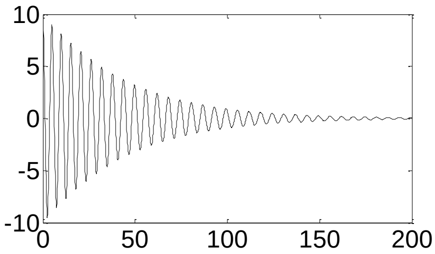


Fig. 1 Displacement response of Duffing system

To verify the influence on synchrosqueezing value $T_x(\omega, b)$, Morlet, Gauss and Bump wavelet is chose to conduct CWT, leading to scalogram as Fig. 2. Synchrosqueezing wavelet transform is then performed to extract IF curves according to CWT of response signals. The identified results are shown in Fig. 3.

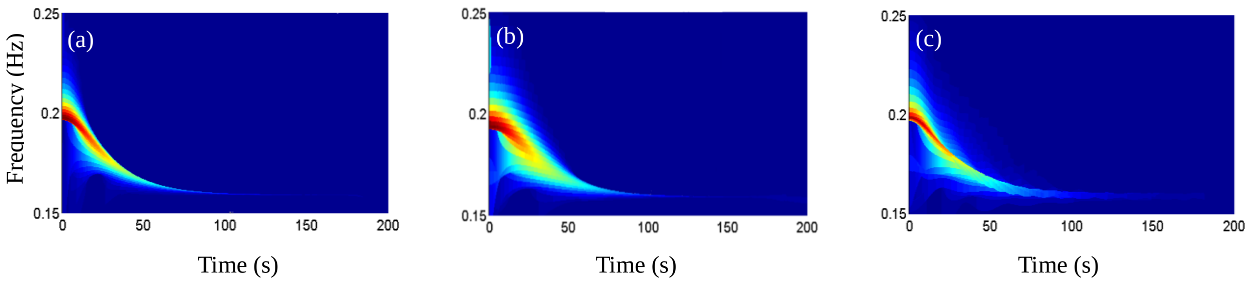


Fig. 2 Wavelet scalogram of displacement response of Duffing system: (aMorlet wavelet, (b)Gauss wavelet, (c)Bump wavelet

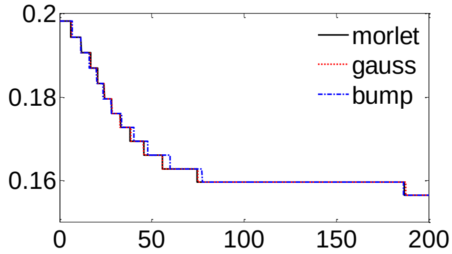


Fig. 3 Identified instantaneous frequency of Duffing system using synchrosqueezing wavelet transform

It can be clearly seen from Fig. 3 that the IF decreases rapidly at the beginning of free vibration, and gradually approaches an asymptotic value (0.16Hz) of the corresponding linear system. This phenomenon also means that cubic stiffness is dominant in large amplitude and then linear behavior stand out, with nonlinear effect gradually becoming unimportant. Fig. 3 also show that the identified IFs by using three different wavelet functions (Morlet, Gauss and Bump) are in good agreement with each other, and have the same changing trend as well. Therefore it is safe to infer that instantaneous frequency is not sensitive to the choice of mother wavelet function in virtue of good stability and reliability of synchrosqueezing method. As for the selection of specific wavelet parameters, further studies are needed to determine whether these parameters have great influence on IFs.

3.2 IF extraction of two-story shear building with forced vibration

Synchrosqueezing is not only appropriate for ideal simulation model but also suitable for multi-degree structural model. In this paper, two-story shear building with forced vibration is adopted to establish the viability and effectiveness of synchrosqueezing wavelet transform. See this engineering model in Fig.4. Related mass, damping coefficients and stiffness are listed in Table 1.

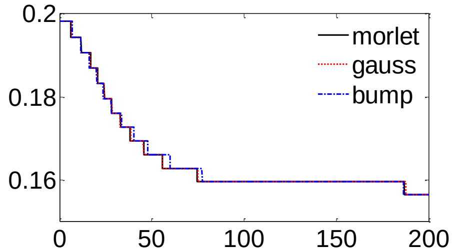


Fig. 3 Identified instantaneous frequency of Duffing system using synchrosqueezing wavelet transform

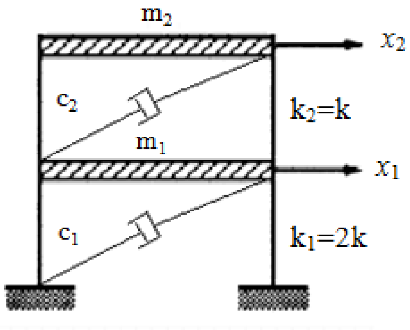


Fig. 4 Two-story shear building

Table 1 Parameters of two-story shear building model

Story	Mass *m* (kg)	damping coefficients *c* (kN·s/m)	Initial stiffness *k* (kN/m)
1st	2.63×10^5	6.95×10^2	2.1×10^5
2nd	1.75×10^5	1.86×10^2	1.05×10^5

The stiffness of the first story k_1 is arbitrary set to be periodically reduced from 2.1×10^5 kN/m to1.4×10^5 kN/m during a period of t=4s and t=16s, that is $k_1 = \left\{2.1 - 0.058(t-4) - 0.131\sin\left[\frac{\pi}{2}(t-4) \times 10^5\mathrm{kN/m}\right]\right\}$. Correspondingly, set the stiffness of the second story k_2 linearly reduces from 1.05×10^5 kN/m to 0.7×10^5 kN/m over a period of t=4s and t=8s, in other words, $k_2 = \{1.05 - 0.0875(t-4) \times 10^5\mathrm{kN/m}\}$. The building is subjected to 1940 El Centro ground motion record and Gaussian white noise with zero mean and 0.1g (gravitational acceleration) standard deviation, and Runge-Kutta algorithm is used to simulate the displacement, velocity and acceleration response of this structural model, with time interval of 0.02s. Fig. 5 shows the displacement responses of the first floor excited by El Centro ground motion and Gaussian white noise.

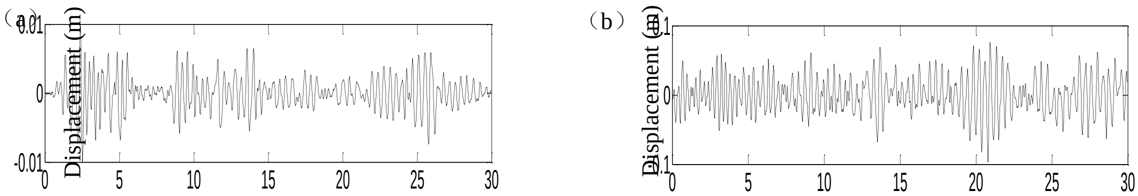


Fig. 5 The first story displacement: (a) El Centro ground motion excitation, (b) Gauss white noise excitation

Three mother wavelet functions mentioned above are used to analyze displacement response of the first floor. For simplicity, only scalogram with Morlet wavelet is shown in Fig. 6. We can see two IF trajectories in it without any difficulty, however, CWT smear the energy of the superimposed IFs around their central frequencies. So the synchrosqueezing wavelet transform is adopted, and the identified results are given in Fig. 7, as well as the exact theoretical results obtained by solving the vibration differential Eq.. Although there are some fluctuations of identified IFs, synchrosqueezing method can still accurately reflect the trend of changing frequency. Comparing 8(a) with 8(b), we can discover that different excitations have different influences on IF identification. Among El Centro ground motion and Gaussian white noise excitation, the latter leads to a better identification. In addition, the results identified by three different wavelet function are quite approximate to theoretical value and don't have distinct differences among themselves, thus the strong stability and robustness for synchrosqueezing algorithm are confirmed once again.

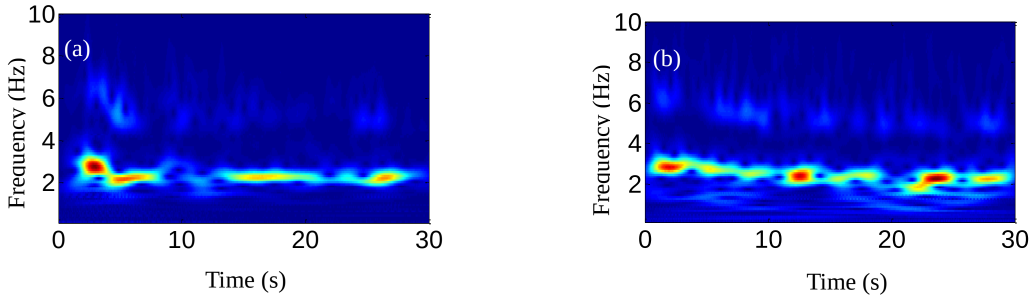


Fig. 6 Wavelet scalogram with Morlet wavelet function: (a) El Centro ground motion excitation, (b) Gaussian white noise excitation

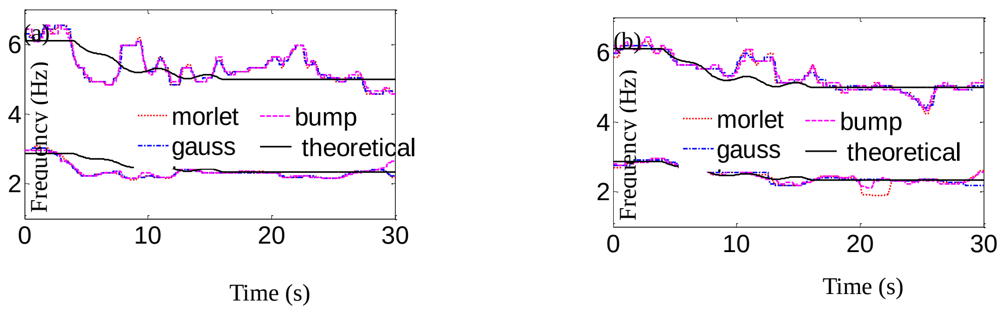


Fig. 7 Identified instantaneous frequency using synchrosqueezing wavelet transform: (a) El Centro earthquake excitation, (b) Gaussian white noise excitation

4. Test verification

To validate the accuracy of synchrosqueezing wavelet transform, a cable structure subjected to a time-varying tension force is considered. The cable is fixed at one end, and the other end is connected to MTS loading system. The accelerometer is arranged at the mid-point of the cable. The total length of cable between two ends is 4.55m. The initial tension force is set to 20 kN, and then the tension force is arbitrary varying with time. At the same time of adjusting cable tension force, hammer was used to generate free vibration and vertical acceleration response was collected with sampling frequency of 600 Hz. The test setup is shown in Fig. 8.

Tension force with linear and sinusoidal change was considered during the test. The theoretical IFs of cable are obtained by solving eigenvalues and eigenvectors of vibration Equation, assuming that parameters keep invariant over a relatively short time interval, which is defined as time frozen method in(Cooper 1990). Owing to the robustness of synchrosqueezing method, complex Morlet wavelet was taken as representative to conduct IF identification. In mathematics, complex Morlet wavelet is a wavelet composed of a complex exponential carrier multiplied by a Gaussian window. The wavelet is defined as

$$\psi_\sigma(t) = c_\sigma \pi^{-\frac{1}{4}} e^{-\frac{t^2}{2}} \left(e^{-i\sigma t} - \kappa_\sigma\right) \tag{11}$$

where $\kappa_\sigma = e^{-\frac{\sigma^2}{2}}$ is defined by the admissibility criterion and the normalisation constant c_σ is:

$$c_\sigma = (1 + e^{-\sigma^2} - 2e^{-\frac{3\sigma^2}{4}})^{-\frac{1}{2}} \tag{12}$$

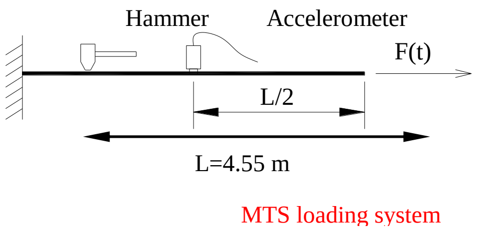
The Fourier transform of the complex Morlet wavelet is

$$\Psi_\sigma(\omega) = c_\sigma \pi^{-\frac{1}{4}} \left(e^{-\frac{(\sigma-\omega)^2}{2}} - \kappa_\sigma e^{-\frac{\omega^2}{2}}\right) \tag{13}$$

The value of σ determines the value of the central frequency F_c and the frequency bandwidth F_b. F_c is the position of the global maximum of $\Psi_\sigma(\omega)$ which, in this case, is given by the solution of the Eq.:

$$(F_c - \sigma)^2 - 1 = \left(F_c{}^2 - 1\right) e^{-\sigma F_c} \tag{14}$$

The parameter σ in the complex Morlet wavelet allows trade between time and frequency resolutions. As a result, we can obtain the optimum resolution by changing the value of σ continuously.



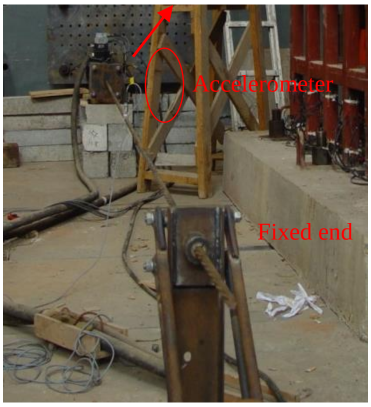


Fig. 8 Cable test setup

4.1 IF identification of cable with linear varying tension force

In this test, the initial tension force of cable is set to 20 kN, and then the tension force increases linearly at the rate of 1.67 kN/s. The setting cable tension force is shown in Fig.9, and measured acceleration response signal is presented in Fig.10. Complex Morlet and synchosqueezing wavelet transform are used to identify cable IFs by analyzing measured acceleration response signal, and the identified results are shown in Fig.11 and Fig. 12, respectively.

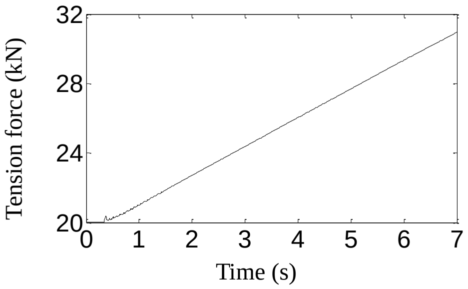


Fig.9 Measured cable tension force with linear variation

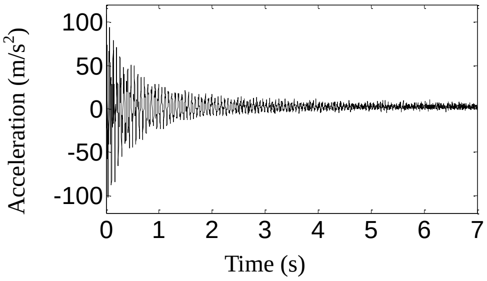


Fig.10 Measured cable acceleration response

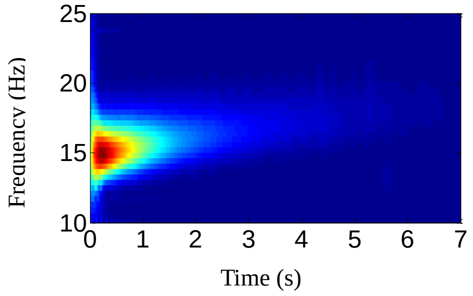


Fig. 11 Wavelet scalogram with Morlet wavelet function

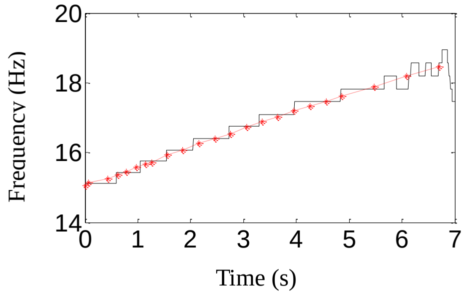


Fig. 12 Identified instantaneous frequency with linearly varying cable tension force

As Fig. 12 shows, the identified IF approximately equal to the theoretical frequency, except at the ending of the vibration. This is mainly because the amplitude of response signal decreases to a low level, which results in a low signal noise ration, and thus affects the result of IF identification to an extent. Moreover, it was clear that end effect remains a long-time problem for wavelet transform, so this disadvantage will inevitably have some impact on identification.

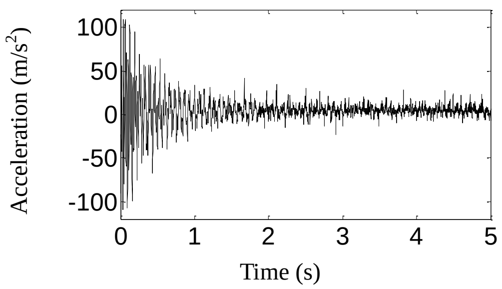


Fig. 13 Measured cable acceleration response with sinusoidally varying tension force

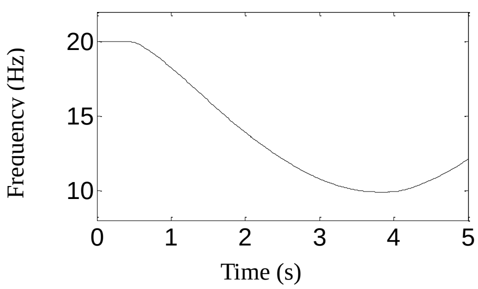


Fig. 14 Measured cable tension force with sinusoidal variation

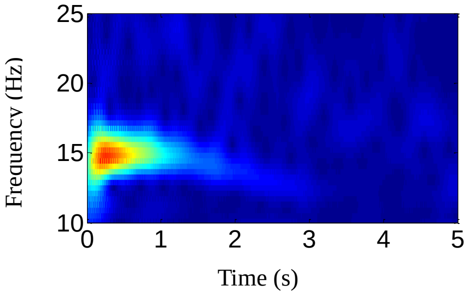


Fig. 15 Wavelet scalogram with Morlet wavelet function

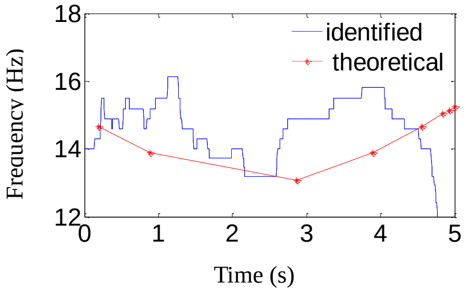


Fig. 16 Identified instantaneous frequency with sinusoidally varying cable tension force

4.2 IF identification of cable with sinusoidal varying tension force

In this condition, the initial tension force of cable is set to 20 kN, and then the tension force varies sinusoidally, which is shown in Fig. 13. The acceleration response signal with duration of 5 seconds was measured at the middle of the cable, shown in Fig. 14. Similar to the mentioned above, Complex Morlet and synchosqueezing wavelet transform are used to estimate cable IFs and the identified results are shown as Fig. 14 and Fig. 15, respectively.

We can see from Fig. 16 that the identified IFs are basically in accordance with theoretical results, and the margin of error is 15%. Due to low signal noise ratio and end effect, synchrosqueezing wavelet transform may encounter the problems of low identification rates and poor recognition effects. Up to now, how to improve the identification accuracy of synchrosqueezing method remains elusive in the field of nonlinear parameter identification, and thus it deserves every effort of study.

5. Conclusion

Sychosqueezing wavelet transform is introduced to directly deal with non-stationary response signals, which is very important feature of structural system in the field of civil engineering. Sychosqueezing algorithm not only clearly describes the relations of changing frequency versus the time , but also enhance the correctness of instantaneous frequency identification. The simulation and test results demonstrate that:

(1) As the name implies, sychosqueezing algorithm can squeeze wavelet

coefficients to refine and sharpen time-frequency curves, and a more accurate instantaneous frequency is obtained as a result. Hence, it is concluded that sychosqueezing wavelet transform can effectively identify the instantaneous frequency of time-varying and nonlinear structure, compared with other standard TFR methods.

(2) Different mother wavelet functions are employed to carry out sensitivity analysis, and the results show that mother wavelet function has a very small impact on instantaneous frequency identification. In a word, the sychosqueezing wavelet transform has great stability and robustness.

(3) Identified instantaneous frequencies are all very close to theoretical results when cable tension force varies linearly or sinusoidally. It is verified that low signal noise ratio and end effect may affect the accuracy of IF identification to an extent.

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