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Modeling of contact between beams with arbitrary cross-sections

Hoontae Jung[1)] *Phill-Seung Lee[2)]

[1),2)] *Department of Mechanical Engineering, Korea Advanced Institute of Science and Technology, Daehak-ro 291, Yuseong-gu, Daejeon 34141, Korea*
[*] *Corresponding author. Tel: +82 42 350 1512; E-mail address:phillseung@kaist.edu*

ABSTRACT

In general, beam-to-beam contact analysis has been possible only for circular cross-section beams. 3D solid elements have been used for contact analysis of general cross-section beams. In this work, the beam-to-beam contact analysis is formulated for beams with arbitrary cross-sections. We use the continuum mechanics based beam elements which have cross-sectional discretization and thus arbitrary cross-sectional shapes can be easily considered (Yoon et. al. 2012). Adopting the point-to-point contact method (Wriggers and Zavarise 1997 and Litewka 2002), some improved contact formulations are suggested. A contact searching algorithm and a contact determination method are proposed.

1. INTRODUCTION

A contact formulation between three-dimensional beams was first proposed by Wriggers and Zavarise (1997). Additional stiffness and reaction forces are considered at the closest point between two beams. This formulation is called as 'point-to-point' contact and has been widely used for the beam-to-beam contact analysis.

The point-to-point contact methods suggested by Wriggers and Zavarise (1997) and Litewka and Wriggers (2002) only consider circular and rectangular cross-section beams, respectively. Most of commercial software supports the beam-to-beam contact analysis between only circular cross-section beams. The beam-to-beam contact analysis for beams with arbitrary cross-section has been performed using three-dimensional solid finite element models.

In this paper, using the continuum mechanics based beam elements, a contact-searching algorithm and a point-to-point contact formulation considering arbitrary cross-section are proposed. The searching algorithm finds the closest points on beam surfaces. Then, it is determined whether a contact has occurred or not at these points. If a contact has occurred, the proposed point-to-point contact formulation are applied at the contact points.

[1)] Ph.D. Student
[2)] Associate Professor

2. CONTINUUM MECHANICS BASED BEAM ELEMENTS

In this section, the continuum mechanics based beam element is briefly presented (Yoon et. al. 2012). The three-dimensional solid finite element can be directly degenerated to a q-node beam element as follows:

$$\vec{x}^{(m)} = \sum_{k=1}^{q} h_k(r) \sum_{j=1}^{p} h_j(s,t)\, \vec{x}_k^{\,j(m)} , \tag{1}$$

where $\vec{x}_k^{\,j(m)}$ is the vector of cross-sectional plane node j of longitudinal solid element m as shown Fig. 1, $h_k(r)$ and $h_j(s,t)$ are the one and two-dimensional shape functions, respectively. Applying the assumption of the Timoshenko beam theory for all the cross-sectional plane nodes in Eq. (1), the geometry interpolation of the continuum mechanics based beam element is given by

$$\vec{x}^{(m)} = \sum_{k=1}^{q} h_k(r)\vec{x}_k + \sum_{k=1}^{q} h_k(r)\bar{y}_k^{(m)} \vec{V}_{\bar{y}}^{k} + \sum_{k=1}^{q} h_k(r)\bar{z}_k^{(m)} \vec{V}_{\bar{z}}^{k} + \sum_{k=1}^{q} h_k(r)\bar{f}_k^{(m)} \alpha_k \vec{V}_{\bar{x}}^{k} , \tag{2}$$

with $\bar{y}_k^{(m)} = \sum_{j=1}^{p} h_j(s,t)\bar{y}_k^{j(m)}$, $\bar{z}_k^{(m)} = \sum_{j=1}^{p} h_j(s,t)\bar{z}_k^{j(m)}$, $\bar{f}_k^{(m)} = \sum_{j=1}^{p} h_j(s,t)\bar{f}_k^{j(m)}$

in which $\bar{y}_k^{(m)}$ and $\bar{z}_k^{(m)}$ are positions on the cross-section on beam node k, $\bar{y}_k^{j(m)}$ and $\bar{z}_k^{j(m)}$ are nodal point j on the cross-section of beam node k, $\bar{f}_k^{j(m)}$ is warping value of nodal point j on the cross-section of beam node k, and α_k is the warping degree of freedom at beam node k (Yoon et. al. 2012 and Yoon and Lee 2014).

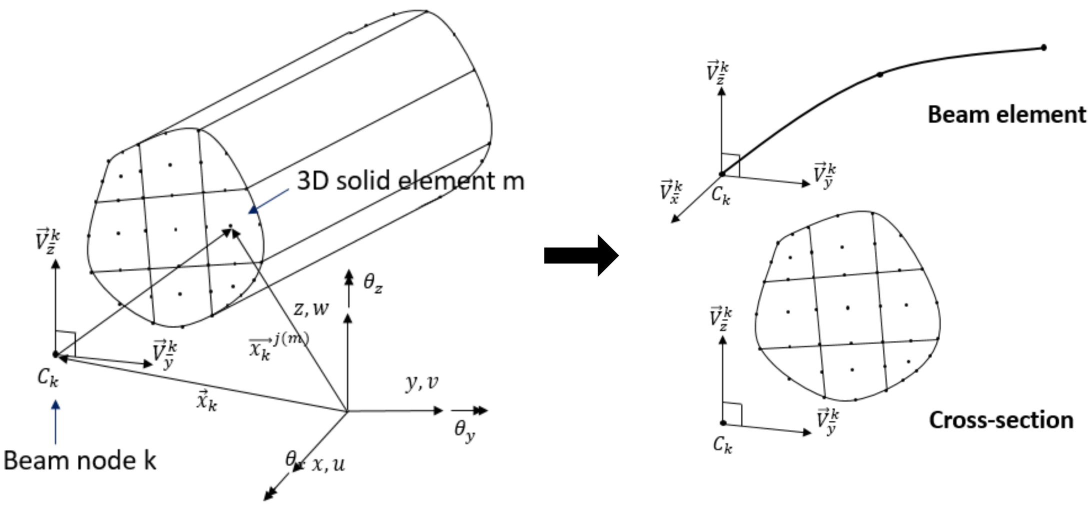

Fig. 1 3D solid finite elements with longitudinal direction and cross-section

3. POINT-TO-POINT CONTACT FORMULATION

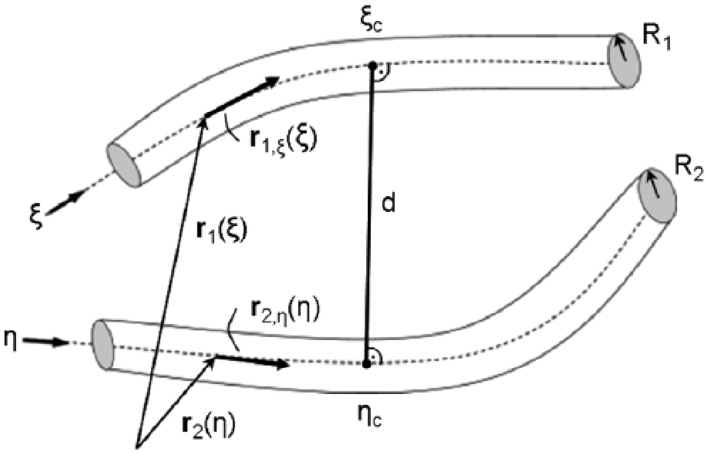

Fig. 2 Kinematics of point-to-point contact of circular cross-section beams (Meier et. al. 2016)

Wriggers and Zavarise (1997) suggested the point-to-point contact formulation between three-dimensional beams with circular cross-section. The contact point is determined by finding minimum distance between center-lines of beams and the distance between center-lines is given by

$$d(\xi,\eta) = \left\| r_1(\xi) - r_2(\eta) \right\| , \tag{3}$$

where ξ and η are parameters that represents the center-lines of beams, see Fig. 2. To determine whether contact has occurred or not, the gap function was introduced as follows:

$$g = d(\xi_c,\eta_c) - R_1 - R_2 , \tag{4}$$

in which R_1 and R_2 are the radii of circular cross-sections of beams, and ξ_c and η_c are parameters that make the distance $d(\xi,\eta)$ in Eq. (3) becomes minimum. If the gap function becomes negative, it is determined that a contact has occurred. When a contact occurs, the following strain energy should be minimized:

$$\Pi = \Pi_{b1} + \Pi_{b2} + \Pi_c , \tag{5}$$

where Π is the total potential energy functional, Π_{b1} and Π_{b2} are the potential energy functionals of two beams, and Π_c is the potential energy functional of contact.

The previous point-to-point contact formulation (Wriggers and Zavarise 1997) can only consider the circular cross-section beam due to the gap function in Eq. (4). To analyze the beam-to-beam contact of arbitrary cross-section beams, we propose improved contact searching algorithm and point-to-point contact formulations.

To determine whether contact of arbitrary cross-section beams has occurred, let us define the distance between surfaces of two beams as follows:

$$d^s\left(r_1, s_1^s, t_1^s, r_2, s_2^s, t_2^s\right) = \left\| \vec{x}_1(r_1, s_1^s, t_1^s) - \vec{x}_2(r_2, s_2^s, t_2^s) \right\|, \tag{6}$$

where $\vec{x}_1$ and $\vec{x}_2$ are the position vectors of two beams shown in Fig. 3, and s_i^s and t_i^s are natural coordinates representing surfaces of beams. First, find 'contact-feasible' points where distance function in Eq. (6) is minimized at current configuration. After updating configuration, verify whether contact-feasible point of a beam belongs to another beam, see Fig. 4. If contact occurs, two beams in contact with each other should minimize the strain energy in Eq. (5). Note that proposed point-to-point contact formulation use the continuum mechanics based beam elements in Eq. (2) and the distance function in Eq. (6) instead of Eq. (3) to calculate the strain energy in Eq. (5).

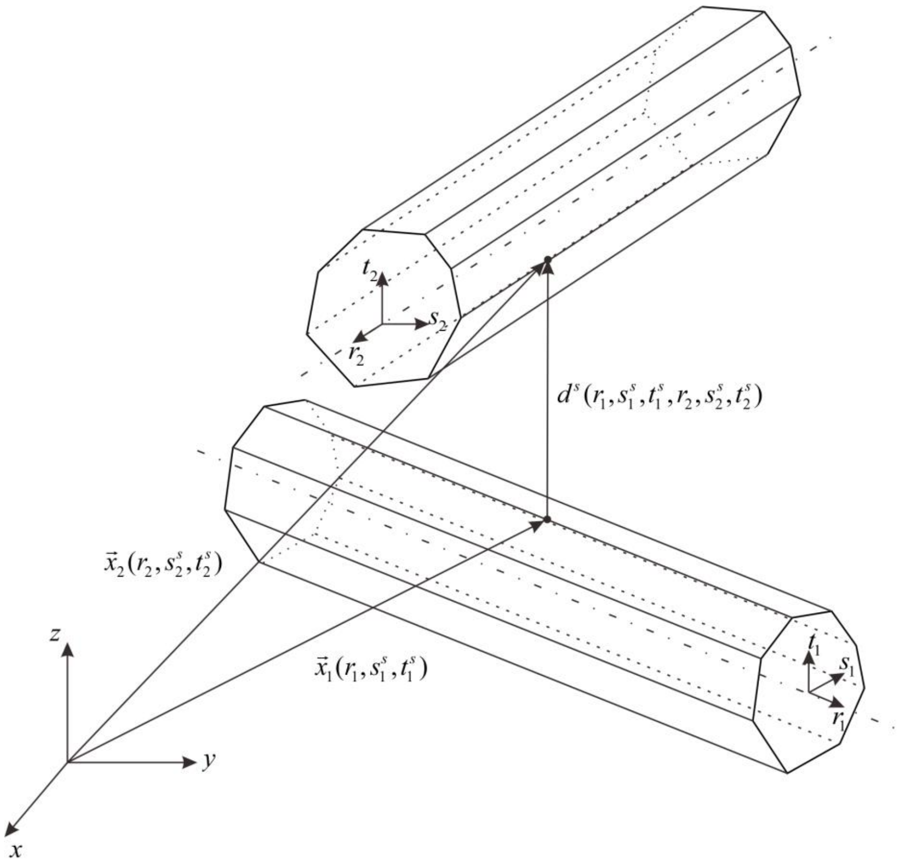

Fig. 3 Kinematics of point-to-point contact of arbitrary cross-section beams

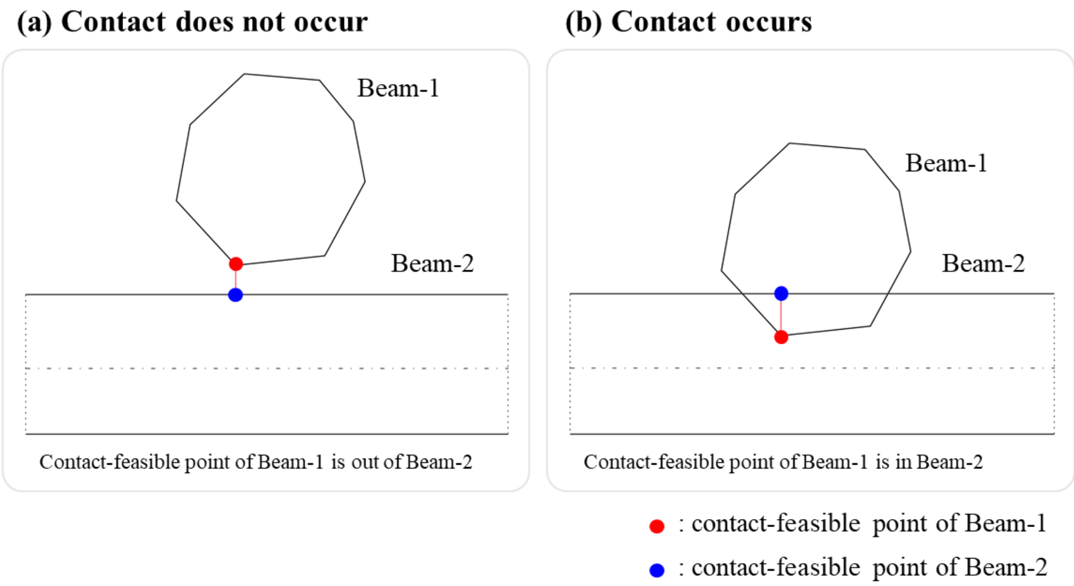

Fig. 4 Description of point-to-point contact of arbitrary cross-section beams

4. NUMERICAL RESULTS

In this section, two cantilever beams with rhombic (diamond shape) cross-section are considered (Litewka and Wriggers 2002). The configuration and properties of the example are shown in Fig. 5 and Table 1. Due to the cross-section shape, contact points only exist at the edge of the beams. We perform the beam-to-beam contact analysis using the proposed point-to-point contact formulation. The results of ANSYS analysis using three-dimensional finite element model are presented for comparison.

Table 2 shows the results of three analyses. The three analysis results of the upper beam, which deforms significantly more than the lower beam, show a good agreement. However, in the case of the lower beam, three results of analyses are significantly different.

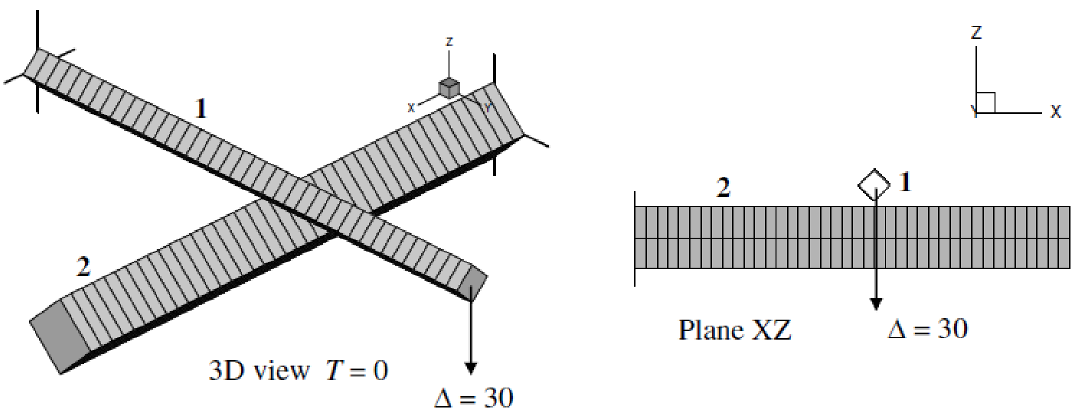

Fig. 5 Two cantilever beams with rhombic cross-section (Litewka and Wriggers 2002)

Table 1 Properties of two rectangular cantilever beams

Upper beam		Lower beam	
E (Young's modulus)	2.0 x 10^4	E (Young's modulus)	3.0 x 10^4
ν (Poisson's ratio)	0.3	ν (Poisson's ratio)	0.17
b (Thickness)	5	b (Thickness)	10
L (Length)	100	L (Length)	100
Initial gap		1.393	
The tip of upper beam is displaced -30 along Z-axis toward 16 elements			

Table 2 Results of contact between rectangular cross-section beams

Previous formulation (Litewka and Wriggers 2002)		ANSYS		Proposed formulation	
	Upper beam		Upper beam		Upper beam
u_x	3.261	u_x	2.1561	u_x	2.314
u_y	-8.260	u_y	-8.3781	u_y	-8.67
u_z	-30	u_z	-30	u_z	-30
	Lower beam		Lower beam		Lower beam
u_x	-0.103	u_x	-0.0968	u_x	-0.07
u_y	-1.331	u_y	-2.2207	u_y	-0.67
u_z	-4.147	u_z	-3.5772	u_z	-3.59

5. CONCLUSIONS

The point-to-point contact formulations proposed by Wriggers and Zavarise (1997) and Litewka and Wriggers (2002) have been widely used to analyse beam-to-beam contact. However, these formulations can only consider beam with circular and rectangular cross-sections. In this paper, point-to-point contact formulation for beam with arbitrary cross-sections is suggested. The continuum mechanics based beam elements which can model arbitrary cross-section with warping effects are used. Searching algorithm for beam with arbitrary cross-section is applied.

The contact problem analysis of rectangular cross section beams is performed. Results of the proposed formulation are compared with the previous formulation and three-dimensional solid element model. Displacement components in the upper beam showed good agreement, but the lower beam results of three analyses cases are significantly different. It is considered that due to the difference of the contact searching algorithm, the contact point and direction of the contact reaction force can be different depending on the contact formulations. In future work, we will consider various beam-to-beam contact cases by using the proposed formulation.

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