

## **Selection of bidirectional ground motions for time history analysis**

\*YAO Hebin<sup>1)</sup>, DONG Yinfeng<sup>2)</sup>, HE Kaiqi<sup>3)</sup> and MA Zhanxiong<sup>4)</sup>

<sup>1), 2)</sup> *Key Laboratory of New Technology for Construction of Cities in Mountain Area (Chongqing University), Ministry of Education, Chongqing, 400045, China*

<sup>1), 2)</sup> *School of Civil Engineering, Chongqing University, Chongqing 400045, China*

<sup>3)</sup> *Southwest Electric Power Design Institute, Sichuan Chengdu 610021, China*

<sup>4)</sup> *Department of Civil Engineering, KAIST, Daejeon 305-600, Korea*

<sup>1)</sup> [dongyinfeng@cqu.edu.cn](mailto:dongyinfeng@cqu.edu.cn), <sup>2)</sup> [409418870@qq.com](mailto:409418870@qq.com), <sup>3)</sup> [263673669@qq.com](mailto:263673669@qq.com)  
<sup>4)</sup> [623723462@qq.com](mailto:623723462@qq.com)

### **ABSTRACT**

When bidirectional ground motions are required for time history analysis, it is commonly to select ground motions in one direction according the requirements of code provisions, and then horizontal counterparts in the perpendicular direction are combined to form bidirectional ground motions. Clearly, this is a compromised method since if ground motions in both directions are selected according to code requirements not enough number of ground motions can be available for time history analysis. This method in which only the conformance between actual and design response spectrum in one direction is assured, usually causes large statistical deviation of structure dynamic response. In this paper, a new selection method is proposed in which ground motions in both directions are selected according to the code requirements and adequate bidirectional ground motions can be selected easily. Using 6 multi-story and tall RC framed building structures as examples, the difference between structure dynamic response to ground motions selected using traditional method and the proposed method is investigated to validate the efficiency of proposed selection method. Besides, the average structure response under 70 bidirectional ground motions is used as a benchmark and the guarantee rates of structure response under bidirectional ground motions of different sample sizes are compared and discussed. Based on the comparison results, the proper sample size of bidirectional ground motions for time history analysis is suggested.

**Key words:** earthquake; time history analysis; bidirectional ground motion; selection; sample size

---

<sup>1)</sup> Graduate student

<sup>2)</sup> Associate professor

<sup>3)</sup> Civil engineer

<sup>4)</sup> Ph.D. candidate

## **1. INTRODUCTION**

In general, the unidirectional earthquake input is mainly considered in the time history analysis of structural seismic response, however there exists a coupling relationship between the seismic response of actual structure in the directions of two principal axes (Yi 2003, Qiu 2001). Even for symmetrical structures, as the elasto-plastic deformation increases or the maldistribution elasto-plastic deformation increases, torsion will also appear in these structures (Yi 2003, Qiu 2001). Therefore, it is more appropriate to consider bidirectional earthquake input when elasto-plastic time history analysis of structures under large earthquakes is performed. In current standards, the selection of earthquake ground motions is majorly based on the principle of spectral match. Several selection methods that are widely used in engineering practice are mainly oriented to the unidirectional earthquake input cases (GB50011-2010, He 2016). For structures that need to consider bidirectional earthquake input, a certain number of unidirectional ground motions are usually selected according to the code specification first, and then these ground motion components and their corresponding orthogonal components are combined to form bidirectional ground motion input. Obviously, this is a simple workaround, but if ground motions in both directions are selected as the code provision requires, it is difficult to select a sufficient number of ground motions for analysis. Moreover, in these methods only the response spectra in one direction is designated to match the design response spectrum. Therefore, it often leads to statistically significant dispersion of structural responses.

In addition, reasonable sample size of ground motions is required in structural time history analysis to ensure more reliable results with relatively less computational effort. A reference value of the input sample size of the elastic time history analysis is provided in the Code for Seismic Design of Buildings (GB50011-2010). Yang Pu. et al. (2000) considered that 3 natural ground motions plus 1 synthetic ground motion (abbreviated as "3+1") is a suitable sample size according to the results of the elasto-plastic analysis. However, these conclusions are based on the case of the unidirectional earthquake input. When the structural elastic or elastoplastic time history analysis are performed under the bidirectional earthquake input, how to select the sample capacity is also worth exploring.

To this point, the paper proposes a bidirectional ground motion selection method. A sufficient number of bidirectional ground motions can be selected conveniently. Taking 6 multi-story and high-rise RC frame structures as example, the effectiveness of the proposed method is verified by comparing the structural responses of conventional and the selected bidirectional earthquake input. In addition, based on the elastic and elasto-plastic seismic responses of 70 ground motion input, the differences in the guaranteed rates of the analysis results for different ground motion sample sizes were compared, and a suggested value of sample size for elastic and elastoplastic time history analysis is given based on the comparison results.

## **2. SELECTION OF BIDIRECTIONAL GROUND MOTIONS**

### *2.1 Selection Method*

The principle of the proposed method is to randomly combine the single direction

ground motions which are selected by using common methods as two-direction ground motion inputs. The advantage of this approach is that the ground motions in both directions are selected to ensure the response spectra of two directions of ground motions are in well agreement with the design response spectra in a statistical sense. The main steps of the method are as follows.

1) Single direction ground motions are firstly selected using current conventional methods. In this paper the dual-band method of by Yang (2000) is adopted. Selection principles are as bellow. In two period range  $[0.1, T_g]$  and  $[T_1-\Delta T_1, T_1+\Delta T_2]$ , the mean relative error between the average response spectrum of selected ground motions and the design response spectrum is not more than 15% where  $\Delta T_1$  and  $\Delta T_2$  are parameters that characterize the periodic change of the first frequency band and  $T_g$  is the characteristic period of the design response spectrum. In general,  $\Delta T_1$  and  $\Delta T_2$  may take values of 0.1~0.2 seconds.

2) In this step, ground motions selected in step 1) are filtered again with site conditions and design seismic groupings considered. The variation of the magnitude  $M$  of the selected earthquake was limited in a range around epicenter distance  $R$ . In the paper, the variation range of magnitude  $M$  and epicentral distance  $R$  is determined according to the work by Xu (2013).

Table 1 Ranges of  $M$  and  $R$  for selection

Fortification intensity / Seismic grouping	$M$	$R$ (km)
intensity 6 / Group 1	[4.0, 6.0]	[10, 50]
intensity 6 / Group 2	[5.0, 7.0]	[11, 80]
intensity 6 / Group 3	>7.0	>80
intensity 7 / Group 1	[5.5, 7.0]	[20, 80]
intensity 7 / Group 2	[6.0, 8.0]	[20, 100]
intensity 7 / Group 3	>7.0	>100
intensity 8 / Group 1	[6.5, 8.0]	[30, 80]
intensity 8 / Group 2	[7.0, 9.0]	[50, 150]
intensity 8 / Group 3	>7.5	>110

3) Randomly combine the selected ground motions, and perform time alignment on each combination of ground motions to ensure the strong motion portion of each pair of ground motions occur at the same time. The alignment principle is to move the midpoint of strong motion portion of two envelope curves to the same time. The alignment process is shown in Figure 1.

4) Calculate the correlation coefficient for each pair of ground motions. The envelope curves of two actual horizontal ground motions are basically the same, therefore when time history analysis is performed the ground motion combination with large correlation coefficient is preferred (Xu 2013).

5) Calculate the correlation coefficient of each pair of ground motions and select the combination with the correlation coefficient less than 0.1.

## 2.2 Comparison of Structure Response Obtained by Using Different Methods

In the paper, six unidirectional eccentric structures with eccentric X-direction (long-axis direction of the structure) are designed by taking the reinforced concrete frame structure as an example: three four-story frame structures DC1.1, DC1.3, and DC1.5; three 12-story frame structure GC1.1, GC1.3, and GC1.5. Here the numbers 1.1, 1.3 and 1.5 torsion irregularity coefficients.

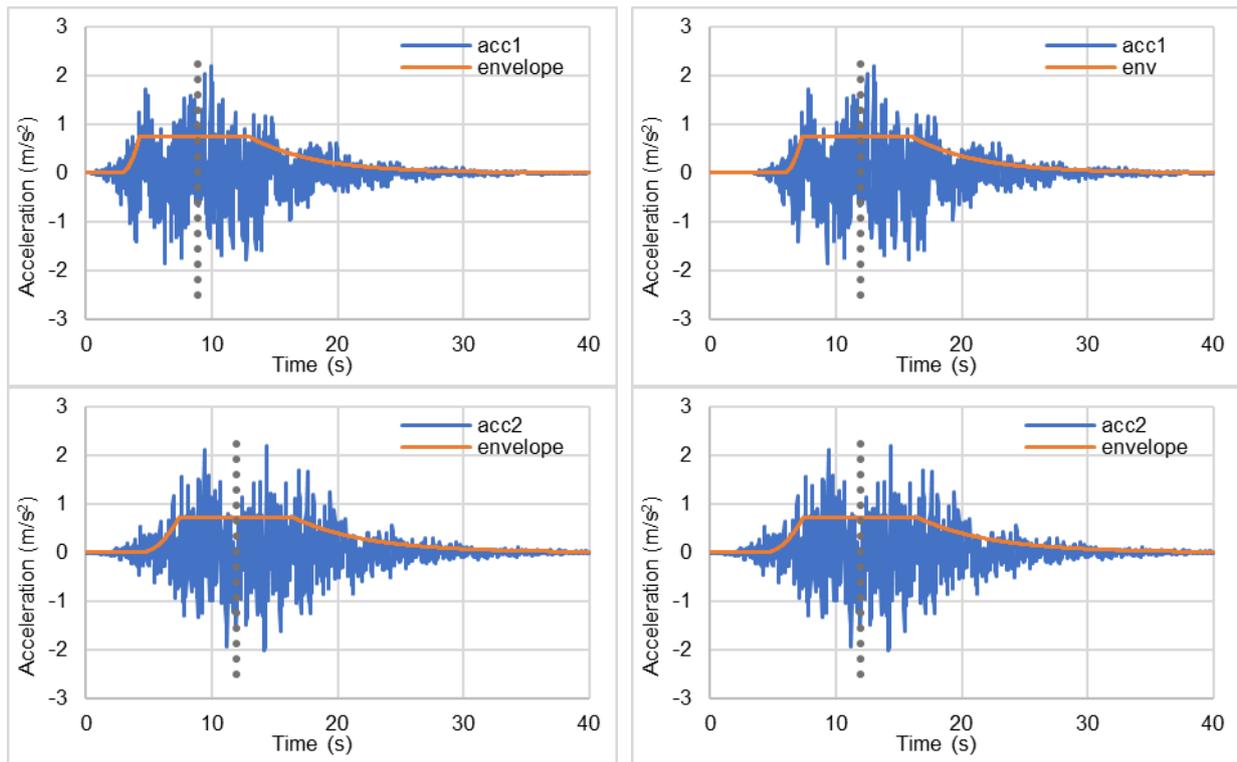
By using common method and the proposed method respectively, 50 natural ground motions are selected to perform elastic and elastoplastic analysis of the DC1.5 structure under bidirectional ground motion input. The mean and coefficient of variation (ratio of standard deviation to mean) of the results, including top displacement, base shear, and story drift rotation, are shown in Table 2 and Fig. 2, where C and B in the legends represent the common method and the proposed method, E and P represent elastic and elasto-plastic response respectively. The results of other structures are similar, so they are not listed here. Since the structure is unidirectionally eccentric in the X direction (longitude direction of the structure), the torsional effect of the structure is mainly reflected the influence of the Y direction (transverse direction of the structure) inputs on X direction structure response, and the X direction input has almost no effect on Y direction structure response. Therefore, only the X direction structure response are compared and discussed in this paper.

It can be seen from Table 2 that both the mean values and coefficients of variation of the top displacement and base shear obtained by the two methods are very close. As for the story drift rotation, the mean values obtained by the two methods in elastic and elasto-plastic cases are basically the same, but the coefficients of variation are significantly different. That is, a smaller variation coefficient of story drift rotation can be obtained by using the proposed method. Story drift rotation can best reflect the damage state of the structures, therefore smaller variation coefficient means better performance of the proposed selection method.

### **3. COMPARISON OF STRUCTURE RESPONSE UNDER UNIDIRECTIONAL AND BIDIRECTIONAL GROUND MOTION INPUTS**

In this section, the differences between the elastic and elastoplastic structure response under unidirectional and bidirectional ground motion inputs are explored. For all the structures, 70 groups of ground motions are used. Considering that story drift rotation is the focus of structural analysis, the mean value and coefficient of variation of the X-direction story drift rotation under different input conditions are compared and analyzed (see Figs. 3~6. In these figures, D and S represent unidirectional and bidirectional ground motion inputs, E and P represent elastic and elastoplastic responses). Mean values are used to compare the difference of structure response under unidirectional and bidirectional ground motion inputs. The coefficients of variation are used to find optimal sample size. In all bidirectional input conditions, X direction is designated as the main input direction, and the peak ground acceleration ratio in the primary and secondary directions is kept 1:0.85.

It can be seen from Figs. 3 and 4 that whether the elastic response or the elastoplastic reaction, high-rise or multi-story structure, the structural response under bidirectional ground motion inputs is larger than the unidirectional input cases, and the structural response increases more obviously as the torsion irregularity coefficients increase. In addition, for elasto-plastic response of multi-story structures, the locations with the maximum story drift rotation under unidirectional and bidirectional inputs are different, which indicates that the bidirectional seismic action may not only amplify the structure response, but also cause the shift of structural damage position.



(a) Before alignment (b) After alignment  
 Fig. 1 Time alignment of ground motions

Table 2 Comparison of base shear and top displacement

	Elastic top displacement		Elastic base shear		Elastoplastic top displacement		Elastoplastic base shear	
	Mean (mm)	Coefficient of variation	Mean (kN)	Coefficient of variation	Mean (mm)	Coefficient of variation	Mean (kN)	Coefficient of variation
Common method	9.2247	0.1339	2786.91	0.1444	57.63	0.1994	9046.35	0.1320
This paper	9.0208	0.1336	2786.96	0.1445	57.05	0.1826	9247.07	0.1318

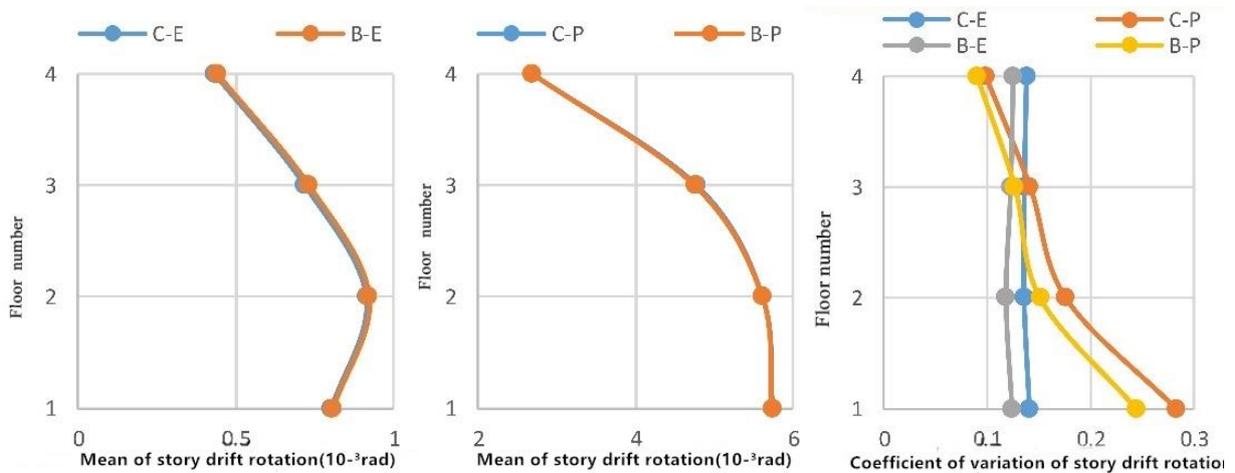
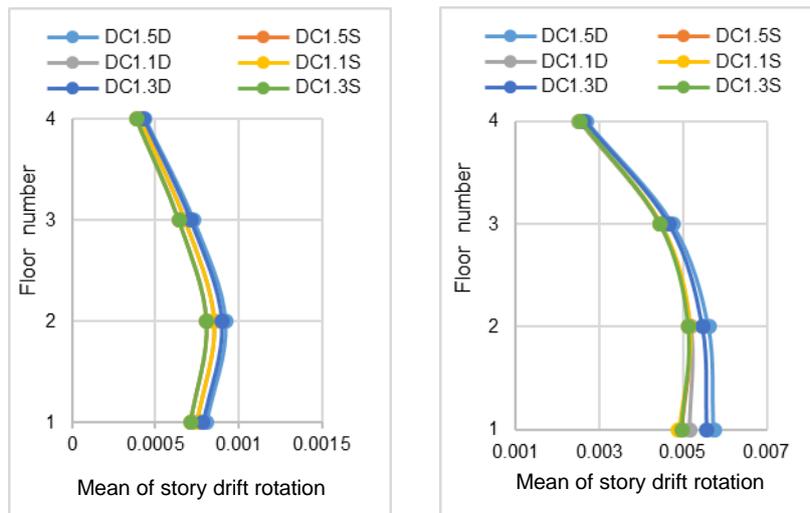


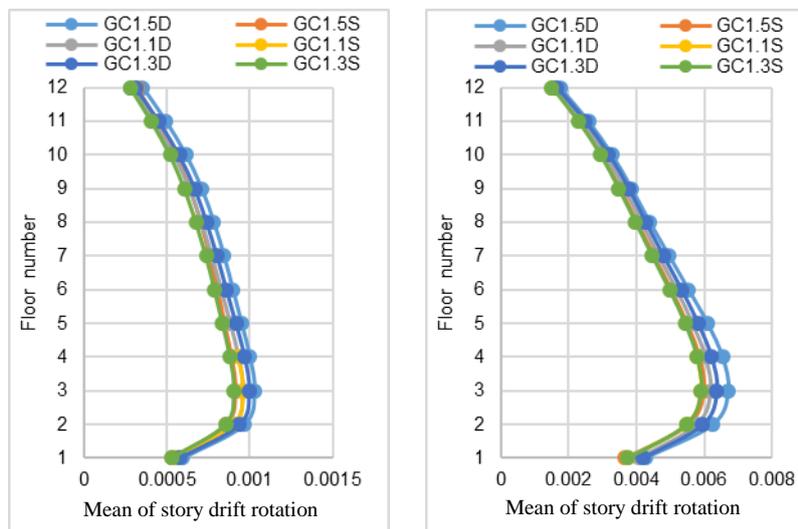
Fig. 2 Comparison of story drift rotation



(a) Elastic

(b) Elastoplastic

Fig. 3 Comparison of story drift rotation of multi-story structures



(a) Elastic

(b) Elastoplastic

Fig. 4 Comparison of story drift rotation of tall building structures

It can be seen from Fig. 5 that for the elastic response of multi-story structures, the coefficients of variation remain basically unchanged and remain stable at around 0.12 along the height of structures, and there is no significant difference between unidirectional inputs and bidirectional inputs. For the elastoplastic response of multi-story structures, the distribution of coefficients of variation from bottom to top is in of an inverted triangle shape with the value gradually decreasing from 0.25 to about 0.1. Based on the results of Figs. 3 and 5, it is shown that the elastoplastic story drift rotation of structures is positively proportional to the corresponding coefficients of variation. With the increase of torsion irregularity coefficients, the difference in the coefficients of variation of elastoplastic story drift rotation under unidirectional and bidirectional inputs increases slightly.

It can be seen from Fig. 6 that for the elastic response of high-rise structures, the

coefficients of variation do not change much in the lower floors and they slightly decrease along the height of structures; in the upper floors of structures, the coefficients of variation slightly increase along the height. In the lower part of structures, the coefficients of variation of story drift rotation under unidirectional and bidirectional inputs are nearly keep constant, but in the upper part the difference in coefficients of variation of story drift rotation under unidirectional and bidirectional inputs increases along the height of structures. The maximum story drift rotation in the lower part is about 2 times that in the upper part, and the maximum coefficient of variation of story drift rotation in the upper part is about 2 times the minimum coefficient of variation in the lower part. The absolute difference in story drift rotation caused by the inputs of the structures is basically evenly distributed along the height.

For the structural elasto-plastic response, the variation coefficients of story drift rotation in the lower show a significant decrease trend as the height of floors increase, and the corresponding value decreases from 0.2 to about 0.1; in the upper part of structures, the coefficients of variation remain stable at about 0.1. From Figs. 3 and 6, it can be inferred that the absolute difference in elastoplastic story drift rotation caused by different inputs is significantly large in the lower part of structures where the absolute difference in elastoplastic story drift rotation is about 6 times that in the upper part of structures. For elastic and elastoplastic response, the absolute difference in structural response caused by different inputs is also obviously large. In the lower part of structures the absolute difference in elastoplastic story drift rotation caused by different inputs is about 6~10 times that in elastic story drift rotation at the same locations, and in the upper part of structures, the ratio is about 2~3 times. Therefore, when performing time history analysis, it is necessary to use different sample sizes ground motion inputs to ensure a sufficient guarantee rate for elastic and elastoplastic analysis cases.

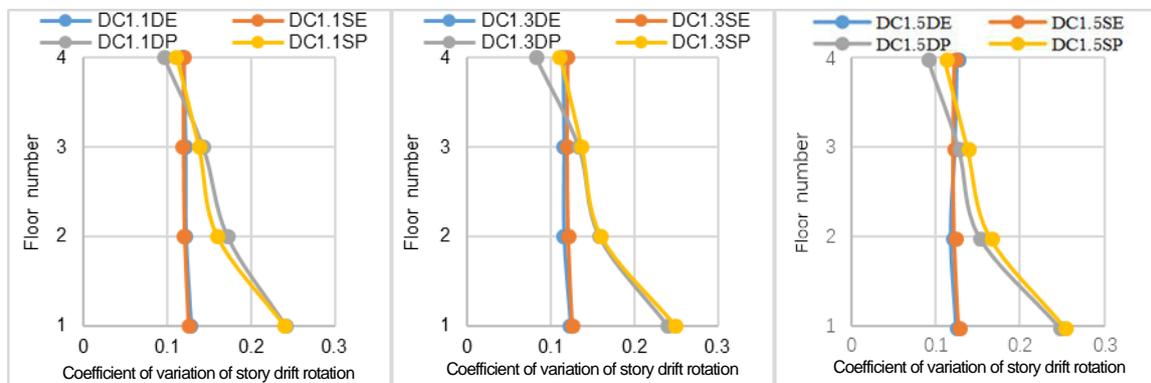


Fig.5 Coefficients of variation of story drift rotation for multi-story structures

#### 4. DETERMINATION OF SAMPLE SIZE

The story drift rotation is an important indicator in structural seismic design. Therefore, when determining the sample size of ground motion inputs, the maximum story drift rotation of the structure is used as a criterion. A large sample size of 55 natural ground motions and 15 synthetic ground motions is used as a benchmark, and 10 small sample size combinations, i.e., “2+1”, “3+1”, “3+2”, “4+2”, “5+2”, “6+3”, “7+3”, “8+4”, “9+4”, and “10+5” are preset for selection. Taking “2+1” combination as an example, here 2 natural

ground motion pairs and 1 synthetic ground motion pair are used with the X direction and Y direction as the major direction respectively, therefore a total of 6 time-history analysis cases (3 cases for each direction) are considered. According to the requirements of code specification, when the sample size is less than 7, the calculation result takes the envelope value, and when the value is greater than 7, the mean of the results is adopted. Corresponding to the elastic and elastoplastic conditions, 4 fluctuation ranges, i.e.,  $\pm 5\%$ ,  $\pm 10\%$ ,  $\pm 15\%$ , and  $\pm 20\%$  of the corresponding analysis results of large sample sizes are used for determining the optimal sample size. For each small sample combination, 10,000 random combinations of ground motion inputs are performed and the probability that analysis results of these combinations fall within the fluctuation range of the large sample analysis results is calculate and used as the guarantee rate for this sample combination case. Taking the structures DC1.3 and GC1.5 as examples, the sample size analysis is performed based on the results of bidirectional inputs and the guaranteed rates under each sample combination of are shown in Tables 3 to 6.

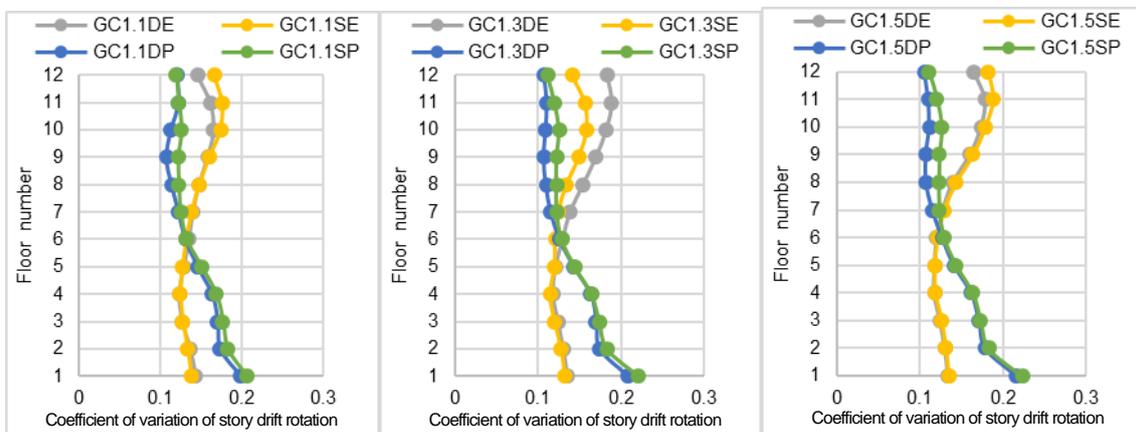


Fig.6 Coefficients of variation of story drift rotation for tall building structures

It can be seen from Tables 3 to 6 that for elastic time history analysis, when the fluctuation range is  $\pm 10\%$  of the large sample analysis result, the guarantee rate of the results is 82.83% for commonly used "2+1" sample combination. For the elastoplastic analysis, when the fluctuation range is  $\pm 10\%$  of large sample analysis results, the guaranteed rate for commonly used "5+2" sample combination is 86.98%. Therefore, it is recommended that the "3+1" sample combination should be selected for the structural elastic time history analysis and the "6+3" sample combination should be used for the structural elastoplastic time history analysis to ensure at least 90% guarantee rate that the deviation between the results of small sample size input and large sample size input is no more than  $\pm 10\%$ .

Table 3 Guaranteed rate of elastic story drift rotation for structure DC1.3

Sample combination	$\pm 20\%$	$\pm 15\%$	$\pm 10\%$	$\pm 5\%$
2+1	99.74%	97.77%	84.65%	53.76%

3+1	99.95%	98.97%	90.16%	58.02%
3+2	100.00%	99.85%	95.46%	67.38%
4+2	100.00%	99.94%	97.22%	71.18%
5+2	100.00%	100.00%	98.30%	74.74%
6+3	100.00%	100.00%	99.60%	82.80%
7+3	100.00%	100.00%	99.73%	85.14%
8+4	100.00%	100.00%	99.98%	90.58%
9+4	100.00%	100.00%	99.99%	92.18%
10+5	100.00%	100.00%	100.00%	95.53%

Table 4 Guaranteed rate of elastic-plastic story drift rotation for structure DC1.3

Sample combination	±20%	±15%	±10%	±5%
2+1	94.41%	84.62%	66.89%	36.96%
3+1	97.38%	88.66%	71.00%	39.97%
3+2	99.45%	95.27%	80.16%	47.61%
4+2	99.67%	96.82%	83.66%	51.24%
5+2	99.93%	98.22%	86.98%	52.96%
6+3	100.00%	99.68%	92.60%	60.42%
7+3	100.00%	99.86%	94.15%	63.88%
8+4	100.00%	99.98%	96.86%	69.60%
9+4	100.00%	100.00%	98.20%	73.27%
10+5	100.00%	100.00%	99.01%	77.47%

Table 5 Guaranteed rate of elastic story drift rotation for structure GC1.5

Sample combination	±20%	±15%	±10%	±5%
2+1	99.88%	97.00%	82.83%	49.25%
3+1	99.99%	98.93%	87.91%	55.30%
3+2	100.00%	99.72%	94.11%	64.02%
4+2	100.00%	99.95%	96.32%	68.68%
5+2	100.00%	99.99%	97.59%	72.60%
6+3	100.00%	100.00%	99.56%	81.60%
7+3	100.00%	100.00%	99.63%	83.75%
8+4	100.00%	100.00%	99.96%	89.05%
9+4	100.00%	100.00%	99.97%	91.45%
10+5	100.00%	100.00%	100.00%	94.46%

Table 6 Guaranteed rate of elastic-plastic story drift rotation for structure GC1.5

Sample combination	±20%	±15%	±10%	±5%
2+1	99.64%	97.54%	85.25%	53.39%
3+1	100.00%	99.24%	90.86%	58.33%
3+2	100.00%	99.63%	94.66%	66.18%
4+2	100.00%	99.96%	97.25%	71.39%
5+2	100.00%	99.98%	98.30%	74.80%
6+3	100.00%	100.00%	99.60%	82.86%
7+3	100.00%	100.00%	99.90%	85.83%
8+4	100.00%	100.00%	99.96%	91.29%
9+4	100.00%	100.00%	100.00%	92.41%
10+5	100.00%	100.00%	100.00%	95.85%

## 5. Conclusions

Selection method and optimal sample size of bidirectional ground motion inputs for time-history analysis are proposed in this paper. The main conclusions are as follows:

1) A bidirectional ground motion selection method is proposed. Using this method, a sufficient number of bidirectional earthquakes can be selected conveniently. Taking 6 multi-story, high-rise reinforced concrete frame structures as an example, the

effectiveness of the proposed method is verified by comparing the structural responses of conventional and the selected bidirectional earthquake input.

2) Using 6 multi-story and tall RC framed building structures as examples, the difference between structure dynamic response to ground motions selected using traditional method and the proposed method is investigated to validate the efficiency of proposed selection method.

3) Besides, the average structure response under 70 bidirectional ground motions is used as a benchmark and the guarantee rates of structure response under bidirectional ground motions of different sample sizes are compared and discussed. Based on the comparison results, the proper sample size of bidirectional ground motions for time history analysis is suggested. It is recommended that the "3+1" sample combination should be selected for the structural elastic time history analysis and the "6+3" sample combination should be used for the structural elastoplastic time history analysis to ensure at least 90% guarantee rate that the deviation between the results of small sample size input and large sample size input is no more than  $\pm 10\%$ .

## Reference

- Yi F.M., Gao X.W., Zhang W.Y., Wang W. (2003), "Response of tall building steel structure to multi-dimensional seismic action", *Journal of Building Structures*, **24**(3). (in Chinese)
- Qiu F.W., Li W.F., Pan P., Qian J.R. (2001), "Quasi-static Test Research of Reinforced Concrete Column Under Biaxial Loading", *Journal of Building Structures*, **22**(5):26-31. (in Chinese)
- GB50011-2010 Code for seismic of building, Beijing: China Architecture & Building Press, 2010. (in Chinese)
- ASCE/SEI 7-10. Minimum Design Loads for Buildings and Other Structures, ASCE, 2010.
- Yang P., Li Y.M., Lai M. (2000), "A new method for selecting inputting waves for time-history analysis", *China Civil Engineering Journal*, **33**(6):33-37. (in Chinese)
- Xu S. (2013), "Selection of rare ground motions for time history analysis of structures", Chongqing: Chongqing University. (in Chinese)
- He K.Q. (2016), "Structure Fragility Analysis under Bidirectional Seismic Action", Chongqing: Chongqing University. (in Chinese)