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# Estimation of Material Properties of a Layered Half-space for Soil-Structure Interaction Analysis

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## ABSTRACT

Material parameters of a layered half-space will be estimated for soil-structure interaction analysis. Dynamic responses of a layered half-space subjected to a harmonic vertical disk load on its surface will be measured and compared with those calculated with estimated material properties of the considered system. The estimated responses will be obtained using the thin-layer method which is very accurate and efficient for a layered medium. An objective function will be defined as $L_2$-norm of difference between the observed and estimated responses. Its gradient will be derived for an application to the considered optimization problem which can estimate material properties of the considered system. The proposed FWI will be applied to verification examples in which higher modes as well as the lowest mode dominate.

## 1. INTRODUCTION

Methods to estimate material properties of an infinite medium have been developed because they can be applied to various problems in related areas. Typical examples of such applications are geophysical subsurface imaging, geotechnical site investigations, and nondestructive tests. The properties can be estimated using full waveform inversion (FWI) (Virieux 2009, Fathi 2015, Mashayekh 2018) or surface wave inversion (SWI) methods (Nazarian 1983, Park 1999, Xu 2006, Astaneh 2016a). The SWI is a very efficient approach to estimate material properties of a layered half-space. However, the application of SWI is not clear for two- and three-dimensional media with irregular material properties and geometries. The FWI will be employed in this study to estimate the material properties of an infinite medium.

The state of the art of the FWI is well summarized in Virieux (2009). In this study, an FWI approach to estimate the material properties of a layered half-space will be formulated. Dynamic responses of a layered half-space subjected to a harmonic vertical disk load on its surface will be measured. They will then be compared with those

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calculated with estimated material properties of the considered system. The estimated responses will be obtained using the thin-layer method, which is very accurate and efficient for a layered medium (Kausel 1981). In the representation, recently developed complex-length finite elements (CFEs) (Astaneh 2016b) and perfectly matched discrete layers (PMDLs) (previously known as continued-fraction absorbing boundary conditions) (Lee 2011) will be considered for an accurate and efficient numerical model for a layered half-space. An objective function will be defined as the $L_2$-norm of the difference between the observed and estimated responses. Its gradient will be derived for application to the considered optimization problem. This approach will allow an estimation of material properties of the considered system. The proposed FWI will be applied to verification examples.

## 2. INVERSE PROBLEM: OBJECTIVE FUNCTION AND ITS GRADIENT

In this study, full waveform inversion (FWI) is considered to estimate the material properties in a layered half-space. For the formulation of FWI, an objective function will be defined in terms of observed and estimated vertical displacements on the surface of a half-space subjected to a harmonic vertical disk load on its surface. The objective function $E(\mathbf{m})$ is defined:

$$E(\mathbf{m}) = \frac{1}{2}\sum_{i=1}^{N_r}\sum_{j=1}^{N_\omega}\left|\tilde{u}_z(r_i,\omega_j) - u_z(r_i,\omega_j)\right|^2 + R(\mathbf{m}) = \frac{1}{2}\sum_{i=1}^{N_r}\sum_{j=1}^{N_\omega}\left|\tilde{u}_{z,i}^{(j)} - u_{z,i}^{(j)}\right|^2 + R(\mathbf{m}) \tag{1}$$

where $\tilde{u}_z(r_i,\omega_j) = \tilde{u}_{z,i}^{(j)}$ is the Fourier transform of the observed or measured vertical displacement at the *i*th measurement due to a harmonic load with a frequency of $\omega_j$, $u_z(r_i,\omega_j) = u_{z,i}^{(j)}$ is the estimated vertical displacement from the thin-layer method, $R(\mathbf{m})$ is a regularization term, and $\mathbf{m}$ is the vector of $N_p$ model parameters to be determined. $N_r$ and $N_\omega$ in Eq. (1) represent the numbers of considered points $r_i$ in the space and frequencies $\omega_j$, respectively. Hereafter, the superscript $(j)$ means that the corresponding quantity is evaluated for a frequency of $\omega_j$. In this study, the material properties and layered structure of a half-space are determined in order to minimize the objective function (1).

A gradient-based optimization technique is employed for the minimization of the objective function $E(\mathbf{m})$. The desired gradient $\partial E/\partial m_k$, where $m_k$ is the *k*th element of the vector $\mathbf{m}$, is given:

$$\frac{\partial E}{\partial m_k} = -\sum_{i=1}^{N_r}\sum_{j=1}^{N_\omega}\mathrm{Re}\left[\left(\frac{\partial u_{z,i}^{(j)}}{\partial m_k}\right)^*\left(\tilde{u}_{z,i}^{(j)} - u_{z,i}^{(j)}\right)\right] + \frac{\partial R}{\partial m_k} \tag{2}$$

In this equation, the superscript $*$ denotes a conjugate of a complex number. The derivative of $\partial u_z/\partial m_k$ can be obtained as follows:

$$\frac{\partial u_z}{\partial m_k} = 2qR\sum_{l=1}^{2N}\phi_{l,z1}\frac{\partial \phi_{l,z1}}{\partial m_k}I_l + qR\sum_{l=1}^{2N}\left(\phi_{l,z1}\right)^2\frac{\partial I_l}{\partial m_k} \tag{3a}$$

$$\frac{\partial I_l}{\partial m_k} = \left(\frac{i\pi}{2k_l}\left[RJ_0\left(k_l r\right)H_2^{(2)}\left(k_l R\right) + rJ_1\left(k_l r\right)H_1^{(2)}\left(k_l R\right)\right] + \frac{2}{Rk_l^3}\right)\cdot\frac{\partial k_l}{\partial m_k} \quad \text{for} \quad 0 \le r \le R \tag{3b}$$

$$\frac{\partial I_l}{\partial m_k} = \frac{i\pi}{2k_l}\left[RJ_2\left(k_l R\right)H_0^{(2)}\left(k_l r\right) + rJ_1\left(k_l R\right)H_1^{(2)}\left(k_l r\right)\right]\cdot\frac{\partial k_l}{\partial m_k} \quad \text{for} \quad r \ge R \tag{3c}$$

Here, $\phi_{l,z1}$ is the element of the *l*th eigenvector $\phi_l$ for the vertical displacement (denoted by the subscript z) on the surface or the first elevation (denoted by the superscript 1) and $k_l$ is the *l*th eigenvalue. The eigenvector and eigenvalue are obtained using the thin-layer method (Kausel 1981). In Eq. (3), $J_n(x)$ is the *n*th-order Bessel function of the first kind, $H_n^{(2)}(x)$ is the *n*th-order Hankel function of the second kind, $R$ and $q$ are the radius and intensity of the harmonic vertical disk load on the half-space surface. The subscript $i$ and superscript $(j)$ are omitted for simple expressions in Eq. (3). It can be observed that the derivatives $\partial\phi_{l,z1}/\partial m_k$ and $\partial k_l/\partial m_k$ of the *l*th eigenvector and eigenvalue are necessary for the gradient of the objective function. They can be obtained using the thin-layer method (Lee 2020).

## 3. APPLICATIONS

The developed FWI approach is applied to estimate of the material properties of layered half-spaces. Their layered structures are shown in Fig. 1. It should be noted that a soft layer is located between two stiff layers in Models 2 and 3. Thus, higher modes dominate their surface responses, and their efficient dispersion curves are combinations of multiple theoretical dispersion curves for the higher modes. On the other hand, the fundamental mode is important for Model 1, and its efficient curve is in good agreement with the lowest theoretical curve.

The profiles of shear-wave velocities are estimated by the proposed FWI. Fig. 1 shows the estimated profiles of the shear-wave velocities. It can be observed that the estimated profiles show satisfactory agreement with the target profiles.

## 4. CONCLUSIONS

In this study, full waveform inversion (FWI) was developed to estimate the material parameters of a layered half-space using the well-established thin-layer method. Vertical displacements of a layered half-space subjected to a harmonic vertical disk load on its surface were measured. The responses were compared with those calculated with estimated material properties of the considered system. The estimated responses were obtained from a thin-layer model for the system. An objective function, defined as the $L_2$-norm of the difference between the measured and estimated responses, was minimized by means of gradient-based optimization to estimate the material properties of the system. The gradient of the objective function was derived in terms of the derivatives of the eigenvectors and eigenvalues of the thin-layer representation of the considered system. The proposed approach was applied to various examples to estimate the profiles of shear-wave velocities of layered half-spaces. It was demonstrated from these applications that the developed FWI can estimate the material properties of layered half-spaces with satisfactory accuracy.

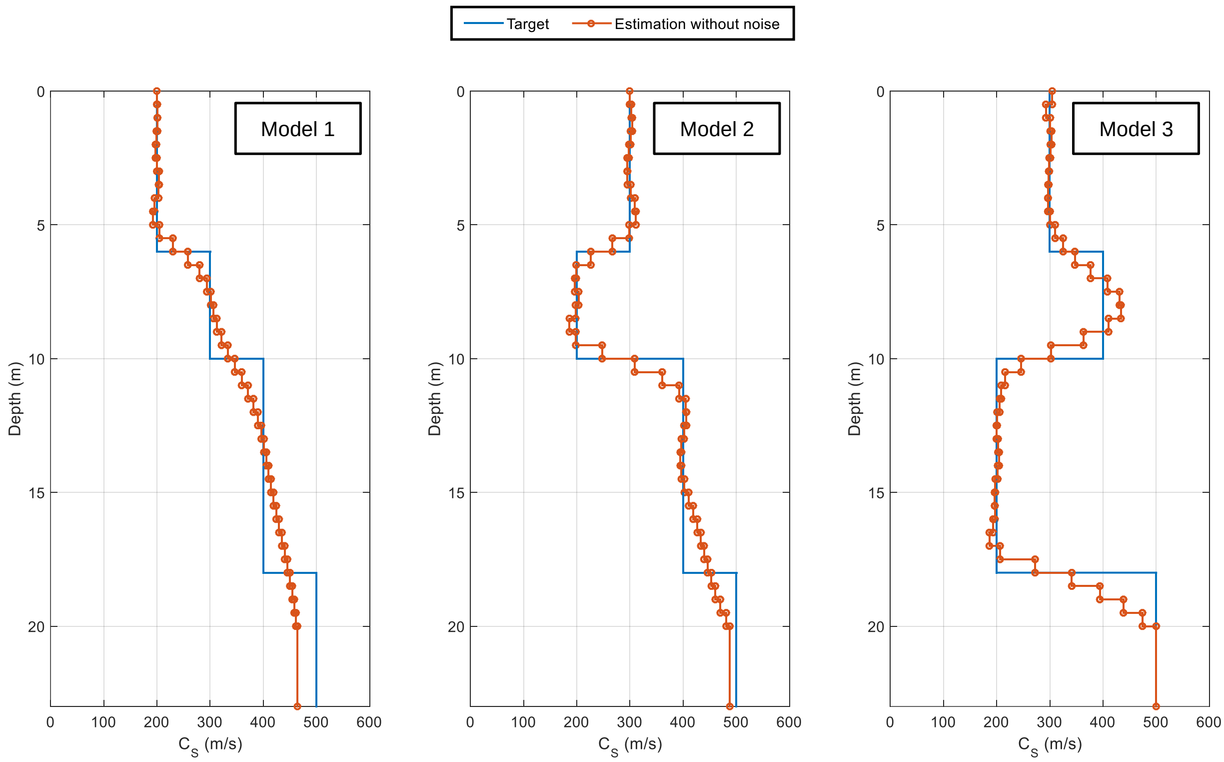


Fig. 1 Estimated profiles of shear-wave velocities in layered half-spaces

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